# Chapter 2: Dosimetric Principles, Quantities and Units

**Set of 131 slides based on the chapter authored by**
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**of the IAEA publication:**
***Review of Radiation Oncology Physics:***
***A Handbook for Teachers and Students***

**Objective:**

To familiarize the student with the basic principles of the quantities used in dosimetry for ionizing radiation.

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**Slide set prepared in 2006**
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## CHAPTER 2. TABLE OF CONTENTS

## 2.1 INTRODUCTION

- Radiation dosimetry has its origin in the medical application of ionizing radiation starting with the discovery of x-rays by Röntgen in 1895.
- In particular
  - **the need of protection against ionizing radiation,**
  - **the application in medicine**

  required quantitative methods to determine a "dose of radiation".
- The purpose of a **quantitative concept of a dose of radiation** is:
  - to predict associated radiation effects (radiation detriments)
  - to reproduce clinical outcomes.

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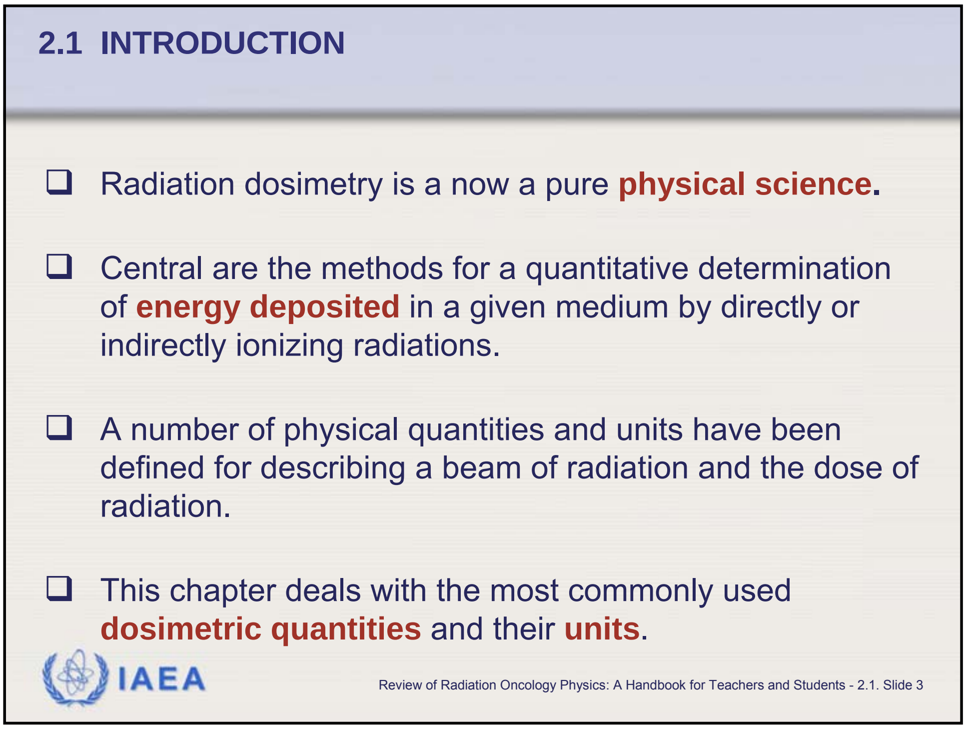
## 2.1 INTRODUCTION

- The connection to the medical profession is obvious.

→ The term **dose of radiation** was initially used in a pharmacological sense, that means: analogously to its meaning when used in prescribing a **dose of medicine**.

- Very soon it turned out that **physical methods** to describe a "dose of radiation" proved superior to any biological methods.



## 2.1 INTRODUCTION

- Radiation dosimetry is a now a pure **physical science.**
- Central are the methods for a quantitative determination of **energy deposited** in a given medium by directly or indirectly ionizing radiations.
- A number of physical quantities and units have been defined for describing a beam of radiation and the dose of radiation.
- This chapter deals with the most commonly used **dosimetric quantities** and their **units**.

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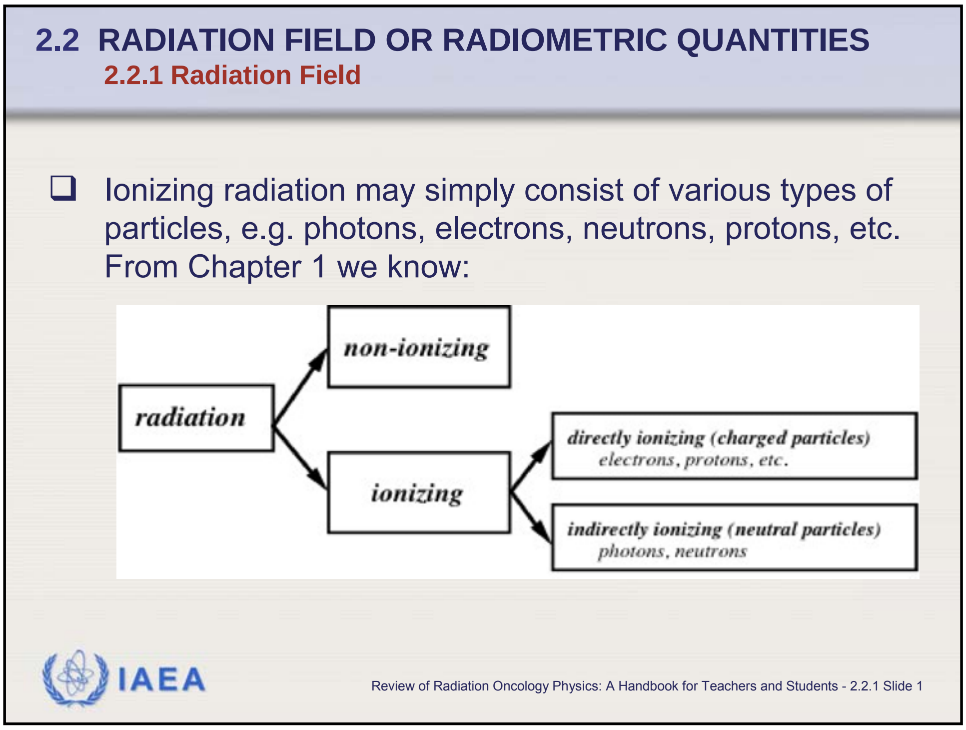

## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.1 Radiation Field

- Ionizing radiation may simply consist of various types of particles, e.g. photons, electrons, neutrons, protons, etc. From Chapter 1 we know:

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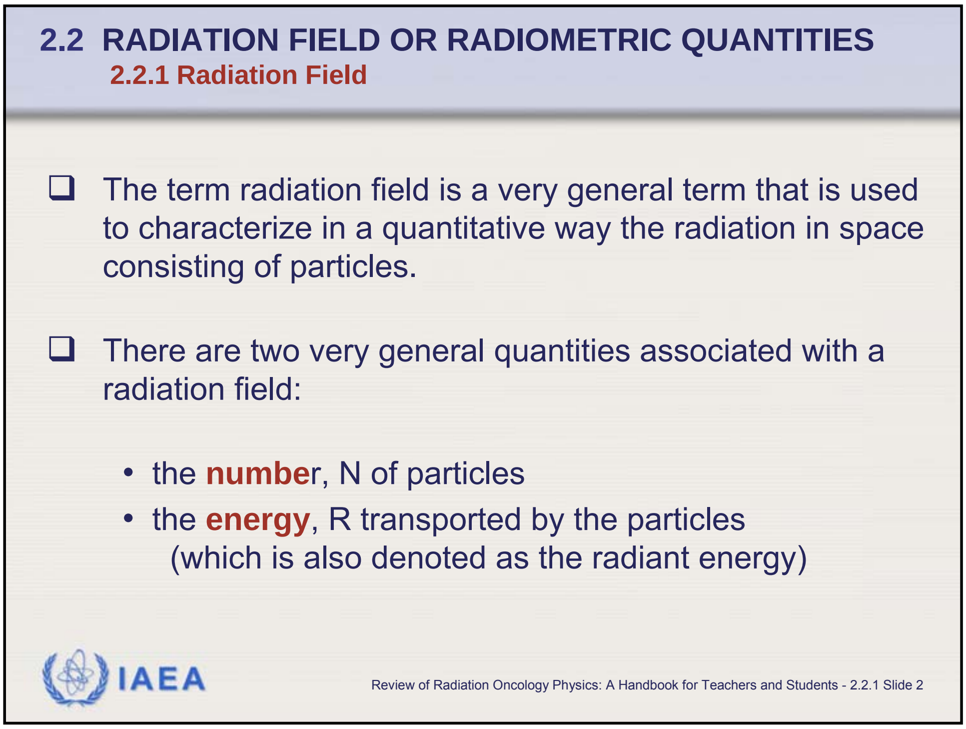
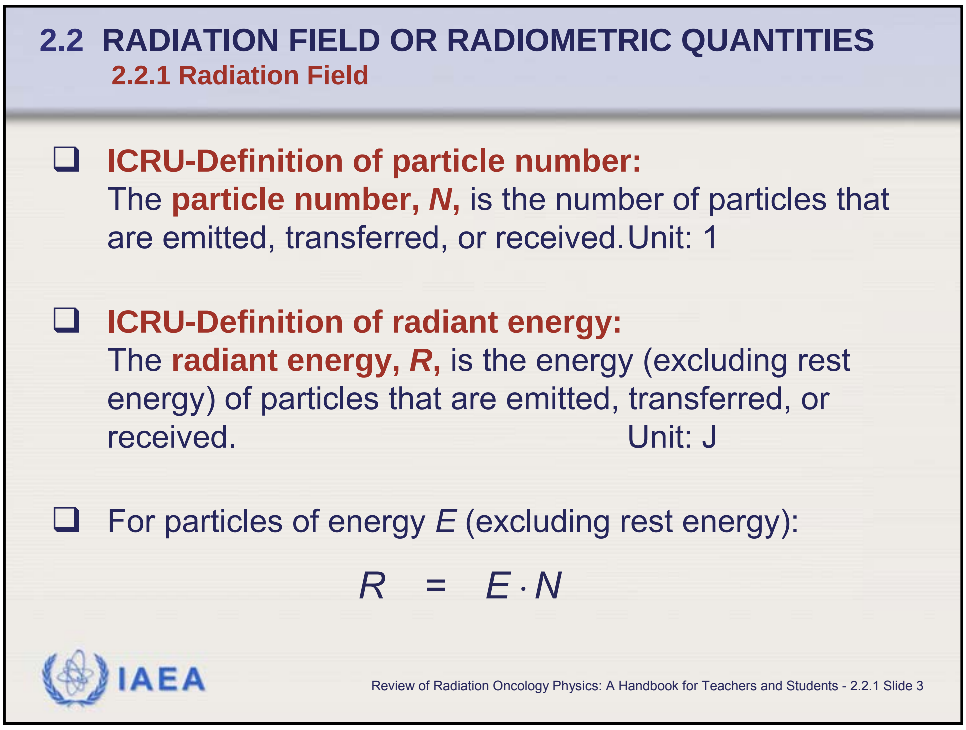


## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.1 Radiation Field

- The term radiation field is a very general term that is used to characterize in a quantitative way the radiation in space consisting of particles.
- There are two very general quantities associated with a radiation field:
  - the **number**, N of particles
  - the **energy**, R transported by the particles (which is also denoted as the radiant energy)



## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.1 Radiation Field

- **ICRU-Definition of particle number:**
  The **particle number, *N*,** is the number of particles that are emitted, transferred, or received. Unit: 1
- **ICRU-Definition of radiant energy:**
  The **radiant energy, *R*,** is the energy (excluding rest energy) of particles that are emitted, transferred, or received. Unit: J
- For particles of energy $E$ (excluding rest energy):

$$R = E \cdot N$$

## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.1 Radiation Field

A **detailed description** of a radiation field generally will require more information on the particle number *N* such as:

- of particle type j
- at a point of interest $\vec{r}$
- at energy E
- at time t
- with movement in direction $\vec{\Omega}$

$$\Rightarrow \quad N = N_j(\vec{r}, E, t, \vec{\Omega})$$

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## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

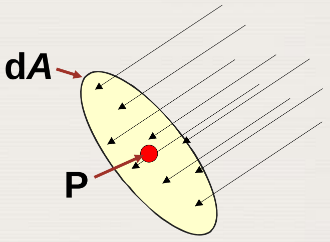
### 2.2.2 Particle Fluence

**How can the number of particles be determined at a certain point in space?**

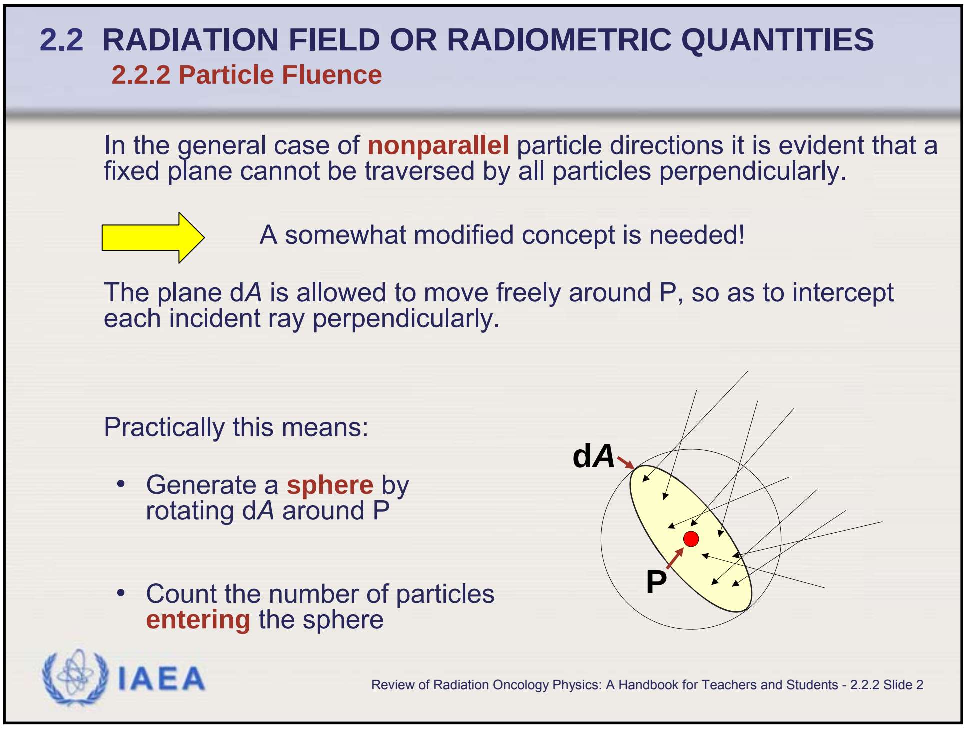
Consider a point P($\vec{r}$) in space within a field of radiation.

Then use the following simple method:

In case of a parallel radiation beam, construct a small area d*A* around the point P in such a way, that its plane is **perpendicular** to the direction of the beam.



Determine the number of particles that intercept this area d*A*.



## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.2 Particle Fluence

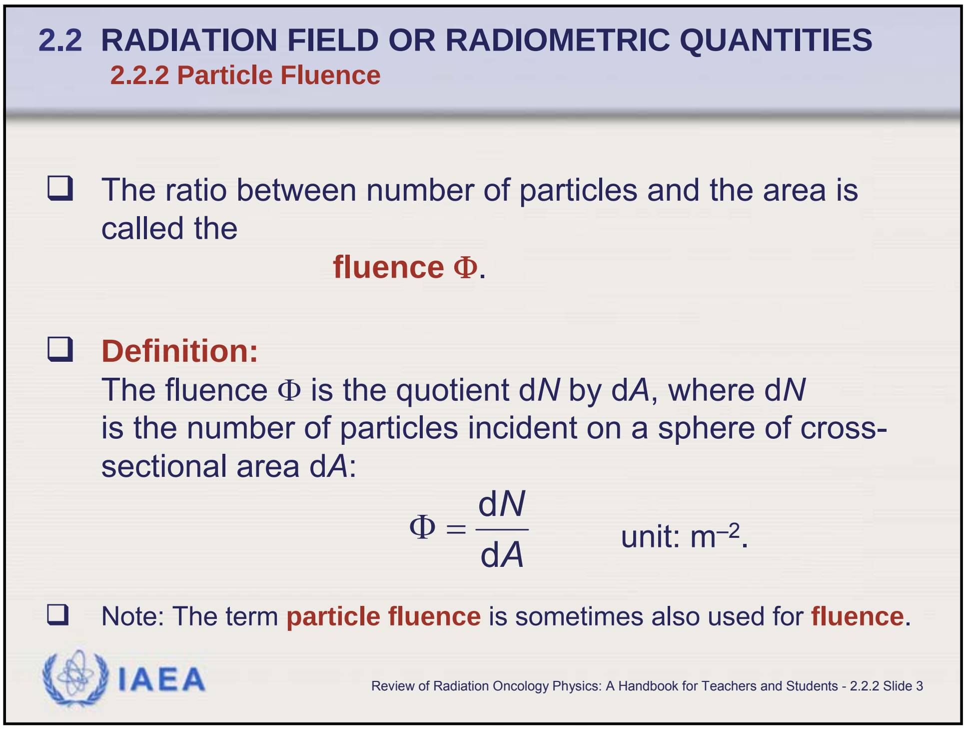
In the general case of **nonparallel** particle directions it is evident that a fixed plane cannot be traversed by all particles perpendicularly.

⇨ A somewhat modified concept is needed!

The plane d*A* is allowed to move freely around P, so as to intercept each incident ray perpendicularly.

Practically this means:

- Generate a **sphere** by rotating d*A* around P
- Count the number of particles **entering** the sphere



## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.2 Particle Fluence

❑ The ratio between number of particles and the area is called the

**fluence $\Phi$**.

❑ **Definition:**
The fluence $\Phi$ is the quotient d*N* by d*A*, where d*N* is the number of particles incident on a sphere of cross-sectional area d*A*:

$$\Phi = \frac{\mathrm{d}N}{\mathrm{d}A} \qquad \text{unit: m}^{-2}.$$

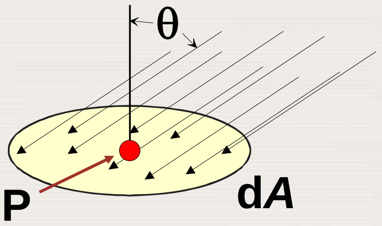
❑ Note: The term **particle fluence** is sometimes also used for **fluence**.

## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.3 Planar Particle Fluence

The definition of **planar particle fluence** refers to the case where the area d$A$ is **not perpendicular** to the beam direction.

- ❑ Planar particle fluence is the number of particles crossing a giving plane per unit area.



- ❑ Planar particle fluence depends on the angle of incidence of the particle beam.

## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.4 Energy Fluence

The same concept used for fluence can be applied to the radiant energy R:

- ❑ **Definition:**
  The energy fluence $\Psi$ is the quotient d$R$ by d$A$, where d$R$ is the radiant energy incident on a sphere of cross-sectional area d$A$:

$$\Psi = \frac{\mathrm{d}R}{\mathrm{d}A}$$

The unit of energy fluence is J $\mathrm{m}^{-2}$.

## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.4 Energy Fluence

Energy fluence can be calculated from particle fluence by using the following relationship:

$$\Psi = \frac{\mathrm{d}N}{\mathrm{d}A} \cdot E = \Phi E$$

where $E$ is the energy of the particle and d$N$ represents the number of particles with energy $E$.

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## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.5 Particle Fluence Spectrum

Almost all realistic photon or particle beams are polyenergetic.

⇨ For a better description, the particle fluence is replaced by the particle fluence **differential** in energy:

$$\Phi_E(E) = \frac{\mathrm{d}^2 N(E)}{\mathrm{d}A \cdot \mathrm{d}E} = \frac{\mathrm{d}\Phi(E)}{\mathrm{d}E}$$

The particle fluence differential in energy is also called the

**particle fluence spectrum.**

## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.6 Energy Fluence Spectrum

The same concept is applied to the radiant energy R:

The energy fluence differential in energy is defined as:

$$\Psi_E(E) = \frac{\mathrm{d}\Psi(E)}{\mathrm{d}E} = \frac{\mathrm{d}\Psi(E)}{\mathrm{d}E} \cdot E$$

The energy fluence differential in energy is also called the

**energy fluence spectrum.**

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## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.6 Energy Fluence Spectrum

**Example of Spectra:**

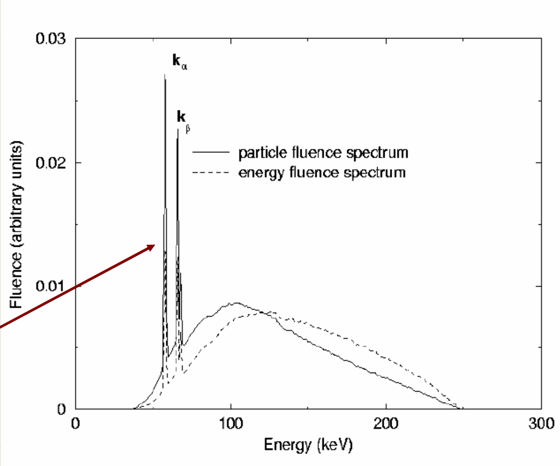
Photon fluence spectrum and energy fluence spectrum generated by an orthovoltage x-ray unit with a $kV_p$ value of 250 kV and an added filtration of 1 mm Al and 1.8 mm Cu.

Target material: tungsten;

Inherent filtration: 2 mm beryllium

**Spectra often show physical phenomena:**

The two spikes superimposed onto the continuous bremsstrahlung spectrum represent the $K_\alpha$ and the $K_\beta$ characteristic x-ray lines produced in the tungsten target.



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## 2.2 RADIATION FIELD OR RADIOMETRIC QUANTITIES

### 2.2.7 Particle Fluence Rate and Energy Fluence Rate

The particle fluence or the energy fluence may change with time.

For a better description of the time dependence, the fluence quantities are replaced by the fluence quantities **differential** in time:

$$\dot{\Phi} = \frac{\mathrm{d}\Phi}{\mathrm{d}t} = \frac{\mathrm{d}^2 N}{\mathrm{d}A \cdot \mathrm{d}t} \qquad \dot{\Psi} = \frac{\mathrm{d}\Psi}{\mathrm{d}t} = \frac{\mathrm{d}^2 R}{\mathrm{d}A \cdot \mathrm{d}t}$$

Unit: $\mathrm{m}^{-2} \cdot \mathrm{s}^{-1}$ Unit: $\mathrm{J\ m}^{-2} \cdot \mathrm{s}^{-1}$

The two fluence quantities differential in time are called the **particle fluence rate** and the **energy fluence rate.** The latter is also referred to as **intensity.**

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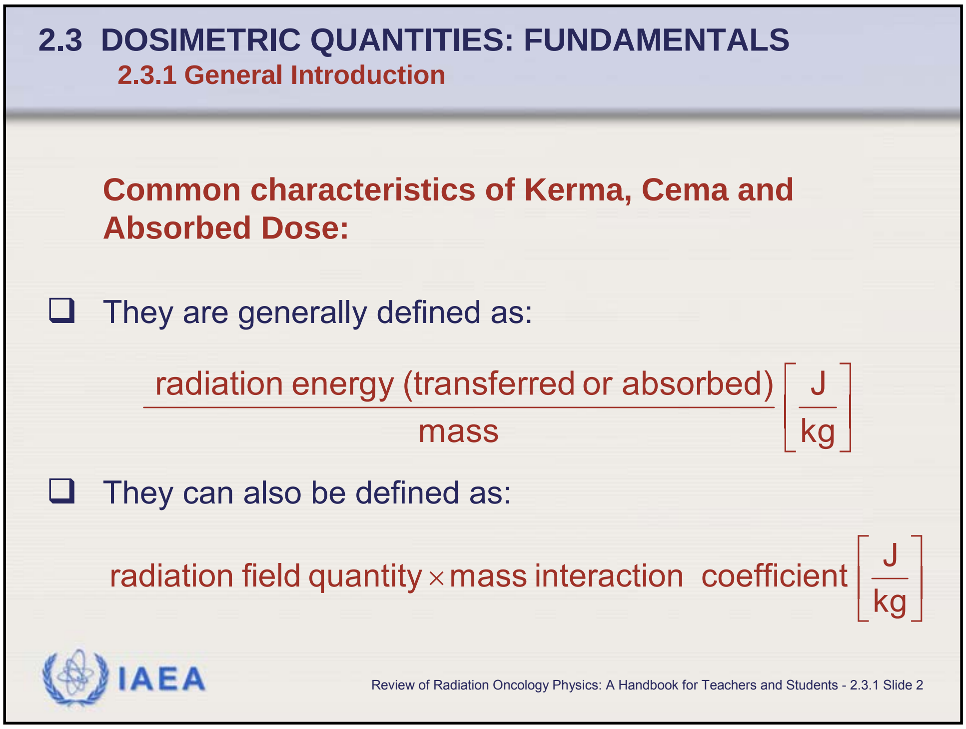
## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.1 General Introduction

The following slides will deal with three dosimetric quantities:

**(1) Kerma**
**(2) Cema**
**(3) Absorbed dose**



## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.1 General Introduction

**Common characteristics of Kerma, Cema and Absorbed Dose:**

- ☐ They are generally defined as:

$$\frac{\text{radiation energy (transferred or absorbed)}}{\text{mass}}\left[\frac{\text{J}}{\text{kg}}\right]$$

- ☐ They can also be defined as:

$$\text{radiation field quantity} \times \text{mass interaction coefficient}\left[\frac{\text{J}}{\text{kg}}\right]$$

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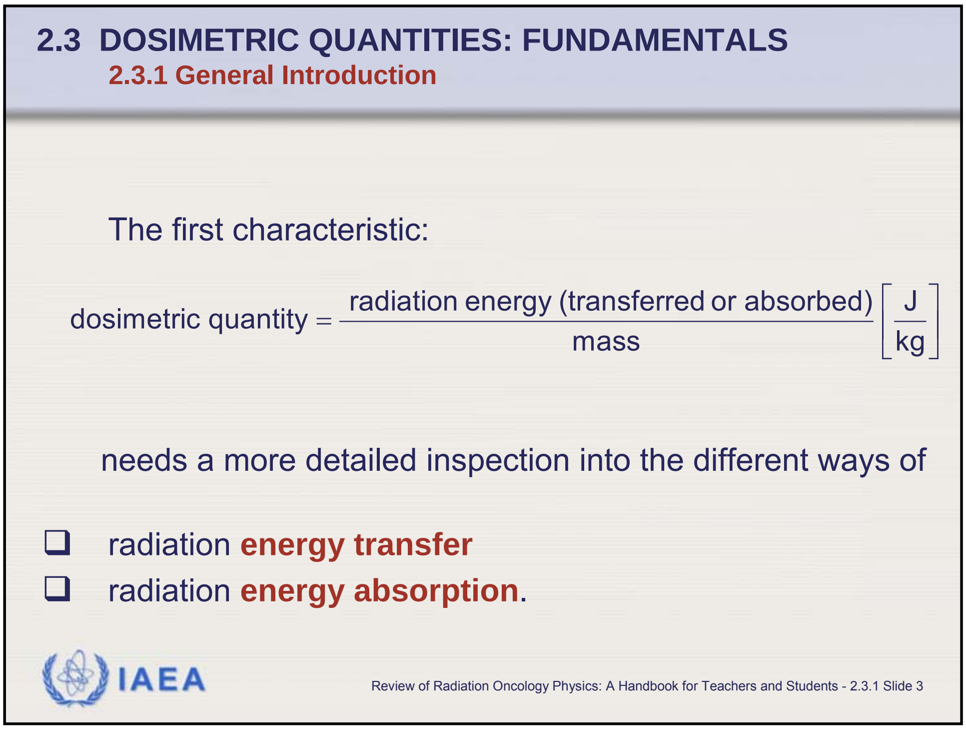

## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.1 General Introduction

The first characteristic:

$$\text{dosimetric quantity} = \frac{\text{radiation energy (transferred or absorbed)}}{\text{mass}}\left[\frac{\text{J}}{\text{kg}}\right]$$

needs a more detailed inspection into the different ways of

- ☐ radiation **energy transfer**
- ☐ radiation **energy absorption**.

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## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.2 Fundamentals of the Absorption of Radiation Energy

Definition of **energy deposit**

- ❑ The term "energy deposit" refers to a **single** interaction process
- ❑ The energy deposit $\varepsilon_i$ is the energy deposited in a single interaction *i*

$$\varepsilon_i = \varepsilon_{in} - \varepsilon_{out} + Q \qquad \text{Unit: J}$$

where

$\varepsilon_{in}$ = the energy of the incident ionizing particle (excluding rest energy),

$\varepsilon_{out}$ = the sum of energies of all ionizing particles leaving the interaction (excluding rest energy),

$Q$ = is the change in the rest energies of the nucleus and of all particles involved in the interaction.

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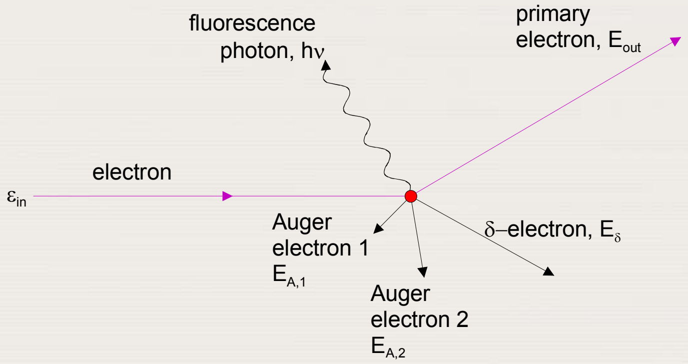
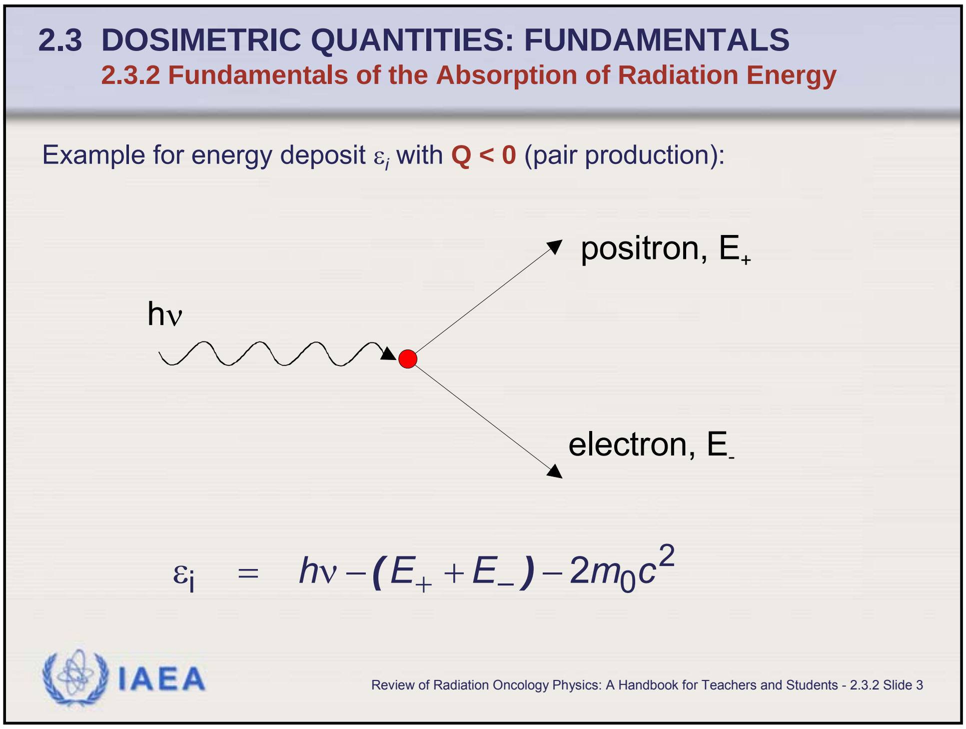
## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.2 Fundamentals of the Absorption of Radiation Energy

Example for energy deposit $\varepsilon_i$ with **$Q$ = 0** (electron knock-on interaction):



$$\varepsilon_i = \varepsilon_{in} - (E_{out} + E_{A,1} + E_{A,2} + E_\delta + h\nu)$$

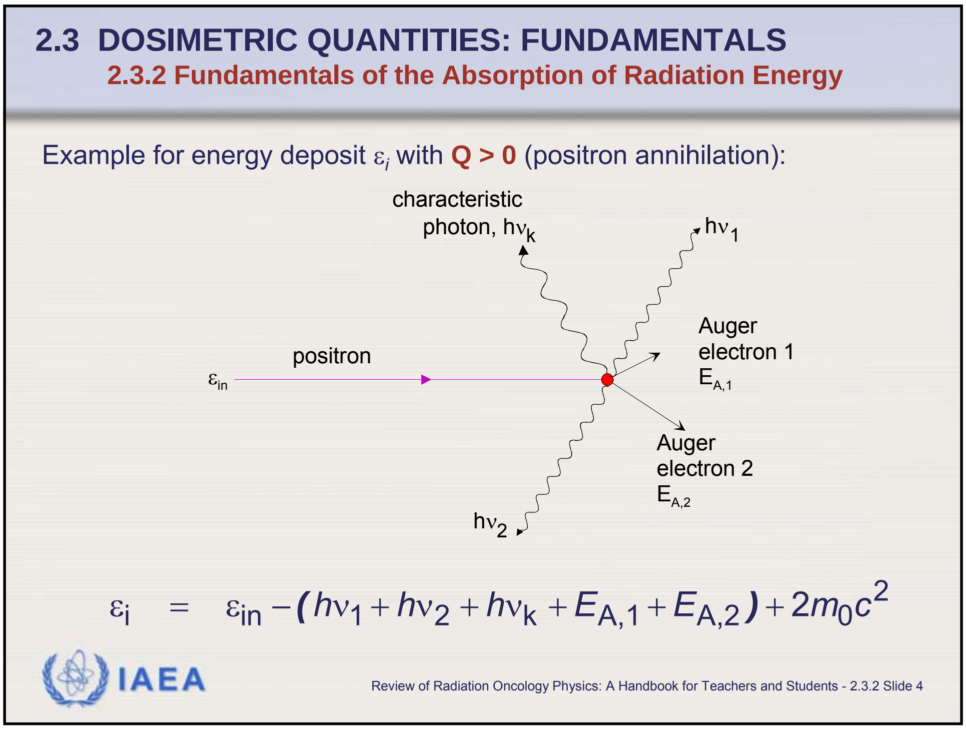


## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.2 Fundamentals of the Absorption of Radiation Energy

Example for energy deposit $\varepsilon_i$ with **Q < 0** (pair production):

$$\varepsilon_i = h\nu - (E_+ + E_-) - 2m_0c^2$$



## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.2 Fundamentals of the Absorption of Radiation Energy

Example for energy deposit $\varepsilon_i$ with **Q > 0** (positron annihilation):

$$\varepsilon_i = \varepsilon_{in} - (h\nu_1 + h\nu_2 + h\nu_k + E_{A,1} + E_{A,2}) + 2m_0c^2$$

## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.2 Fundamentals of the Absorption of Radiation Energy

Definition of **energy imparted**

- ❑ The term "energy imparted" refers to a **small volume**.
- ❑ The energy imparted, $\varepsilon$, to matter in a given volume is the **sum of all energy deposits** in the volume, i.e. the sum of energy imparted in all those basic interaction processes which have occurred in the volume during a time interval considered:

$$\varepsilon = \sum_{i} \varepsilon_i$$

where the summation is performed over all energy deposits $\varepsilon_i$ in that volume.

- ❑ Example: A radiation detector responds to irradiation with a signal $M$ which is basically related to the energy imparted $\varepsilon$ in the detector volume.

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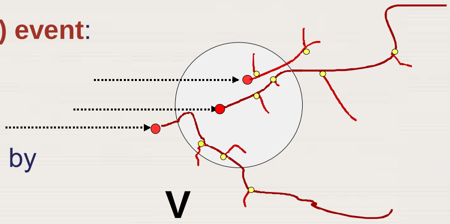
## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.2 Fundamentals of the Absorption of Radiation Energy

Definition of an **(energy impartion) event**:



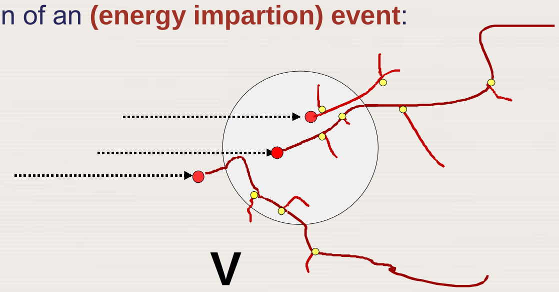
Consider the energy imparted in a volume V by secondary electrons which are generated by primary photons.

- ❑ The incoming **primary photons** are statistically **uncorrelated**.
- ❑ The secondary electrons generated by different photons are **uncorrelated**.
- ❑ However, there is a correlation:
When a particular secondary electron is slowing down, it creates further secondary electrons. The **primary generating photon**, the **generated electron** and **all further electrons** (all generations) are **correlated**.

## 2.3 DOSIMETRIC QUANTITIE: FUNDAMENTALS

### 2.3.2 Fundamentals of the Absorption of Radiation Energy

Definition of an **(energy impartion) event**:



- ☐ Therefore, all single energy deposits:
  - that are caused from an initially generated secondary electron, and
  - that from all further generations of secondary electrons

are correlated in time.

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## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.2 Fundamentals of the Absorption of Radiation Energy

Definition of an **(energy impartion) event**:

- ☐ The imparted energy from statistically correlated particles can be put together.
- ☐ The term "event" was introduced to denote the imparting of energy ε by those statistically correlated energy deposits:

$$\varepsilon = \sum_{j=1}^{N}\left(\sum_{i=1}^{n_j}\varepsilon_i\right)$$

where

**N = number of events**

$n_j$ = number of energy deposits at event j

individual events different in size

Note: The **same amount** of imparted energy $\varepsilon$ can consist of:
- a small number of events each with a large size
- a high number of events each with a small size

## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.3 Stochastic of Energy Absorption

- ☐ Since all energy deposits $\varepsilon_i$ are of stochastic nature, $\varepsilon$ is also a stochastic quantity, the values which follow a probability distribution!

**Stochastic of Energy Absorption** means:
The energy imparted is always **statistically distributed** during the time interval considered. The distribution comes from two sources:

- ☐ fluctuation in the number of events
- ☐ fluctuations in the size of events

➡ The determination of the variance of energy absorption must take into account these two sources!

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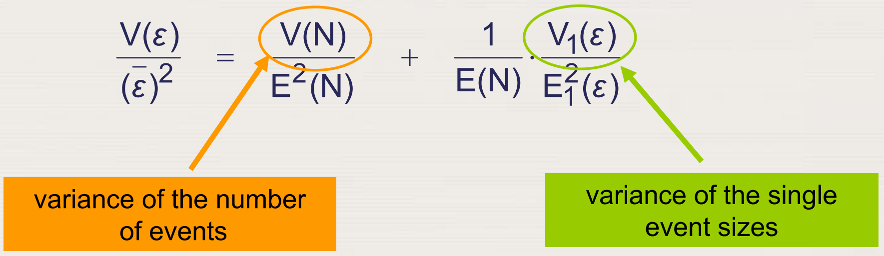
## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.3 Stochastic of Energy Absorption

The combined relative variance of energy imparted $\varepsilon$ is given by:

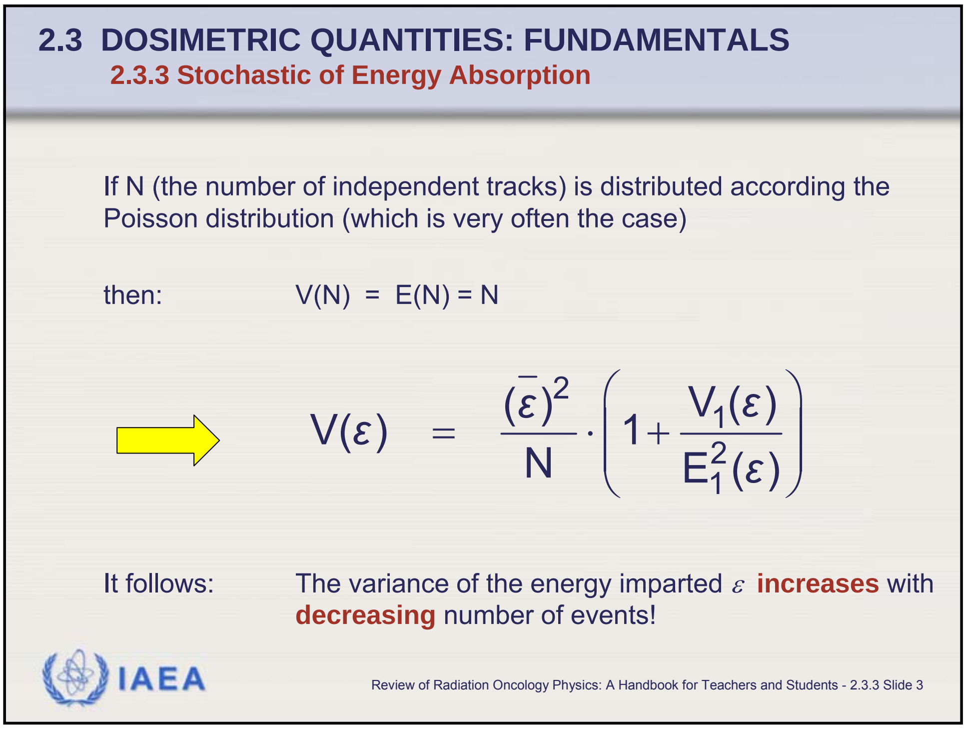


$$\frac{V(\varepsilon)}{(\bar{\varepsilon})^2} = \frac{V(N)}{E^2(N)} + \frac{1}{E(N)} \cdot \frac{V_1(\varepsilon)}{E_1^2(\varepsilon)}$$

variance of the number of events

variance of the single event sizes

where: E = expectation value $E_1$ = single event exp. value
N = number of events $\varepsilon$ = energy imparted



## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

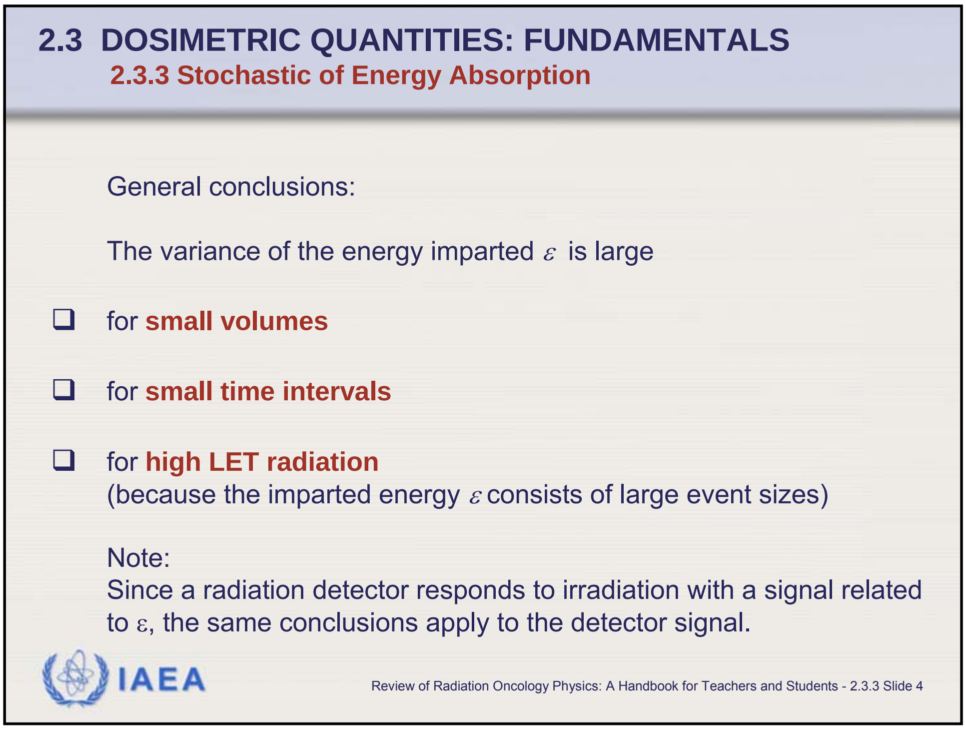
### 2.3.3 Stochastic of Energy Absorption

If N (the number of independent tracks) is distributed according the Poisson distribution (which is very often the case)

then: $V(N) = E(N) = N$

$$\Rightarrow \quad V(\varepsilon) = \frac{(\bar{\varepsilon})^2}{N} \cdot \left(1 + \frac{V_1(\varepsilon)}{E_1^2(\varepsilon)}\right)$$

It follows: The variance of the energy imparted $\varepsilon$ **increases** with **decreasing** number of events!



## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.3 Stochastic of Energy Absorption

General conclusions:

The variance of the energy imparted $\varepsilon$ is large

- for **small volumes**
- for **small time intervals**
- for **high LET radiation**
  (because the imparted energy $\varepsilon$ consists of large event sizes)

Note:
Since a radiation detector responds to irradiation with a signal related to $\varepsilon$, the same conclusions apply to the detector signal.

## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.4 Energy Absorption and Energy Transfer

What is the exact meaning of **"energy absorption "** ?

The term energy absorption refers to **charged particles**, e.g. electrons, protons etc.

From Chapter 1 we know:

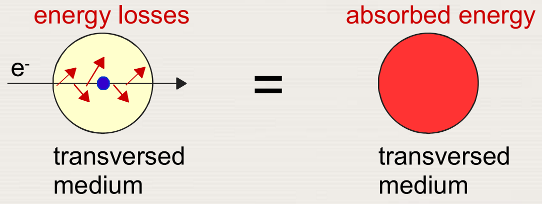
- ❑ Inelastic collisions between an incident electron and an orbital electron are Coulomb interactions result in:
  - ***Atomic ionization*:**
    Ejection of the orbital electron from the absorber atom.
  - ***Atomic excitation*:**
    Transfer of an atomic orbital electron from one allowed orbit (shell) to a higher level allowed orbit
- ❑ Atomic ionizations and excitations result in collision **energy** losses experienced by the incident electron and are characterized by ***collision (ionization) stopping power***.

## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

### 2.3.4 Energy Absorption and Energy Transfer

Continued: What is the exact meaning of **"energy absorption "** ?

- ❑ The **loss of energy** experienced by the incident electron by a collision is at the same time **absorbed** by the absorber atom and thus by a medium.



- ❑ For charged particles, the process of energy absorption in a medium is therefore described by the process of the collision energy loss (the collision stopping power).

## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS
### 2.3.4 Energy Absorption and Energy Transfer

What is the exact meaning of **"Energy Transfer "** ?

The term energy transfer refers to **uncharged particles**, e.g. photons, neutrons etc.

From Chapter 1 we know:

- ☐ The photon fate after an interaction with an atom includes two possible outcomes:
  - *Photon disappears* (*i.e.*, is absorbed completely) and a portion of its energy is transferred to light charged particles (electrons and positrons in the absorbing medium).
  - *Photon is scattered* and two outcomes are possible:
    - The resulting photon has the same energy as the incident photon and no light charged particles are released in the interaction.
    - The resulting scattered photon has a lower energy than the incident photon and the energy excess is transferred to a light charged particle (electron).

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## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS
### 2.3.4 Energy Absorption and Energy Transfer

Continued: What is the exact meaning of **"Energy Transfer "** ?

- ☐ The energy that is transferred in an photon interaction to a light charged particle (mostly a secondary electron) is called an energy transfer.
- ☐ This process is described by the **energy transfer coefficient**

$$\mu_{tr} = \mu \frac{\bar{E}_{tr}}{h\nu}$$

with $\bar{E}_{tr}$ the average energy transferred from the primary photon with energy $h\nu$ to kinetic energy of charged particles ($e^-$ and $e^+$).

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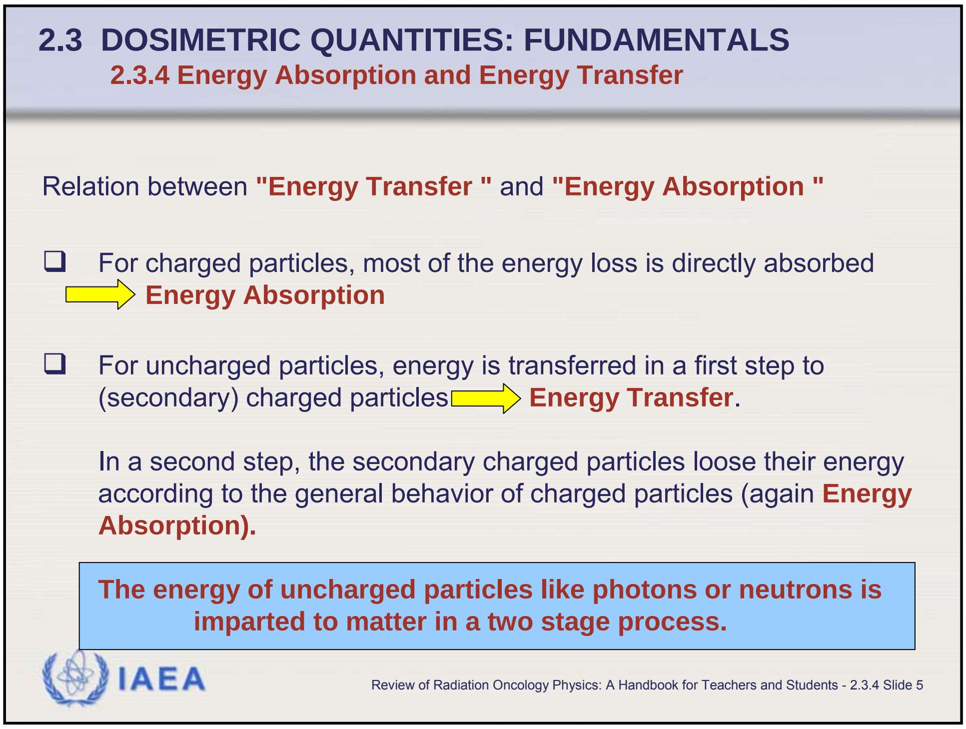


## 2.3 DOSIMETRIC QUANTITIES: FUNDAMENTALS

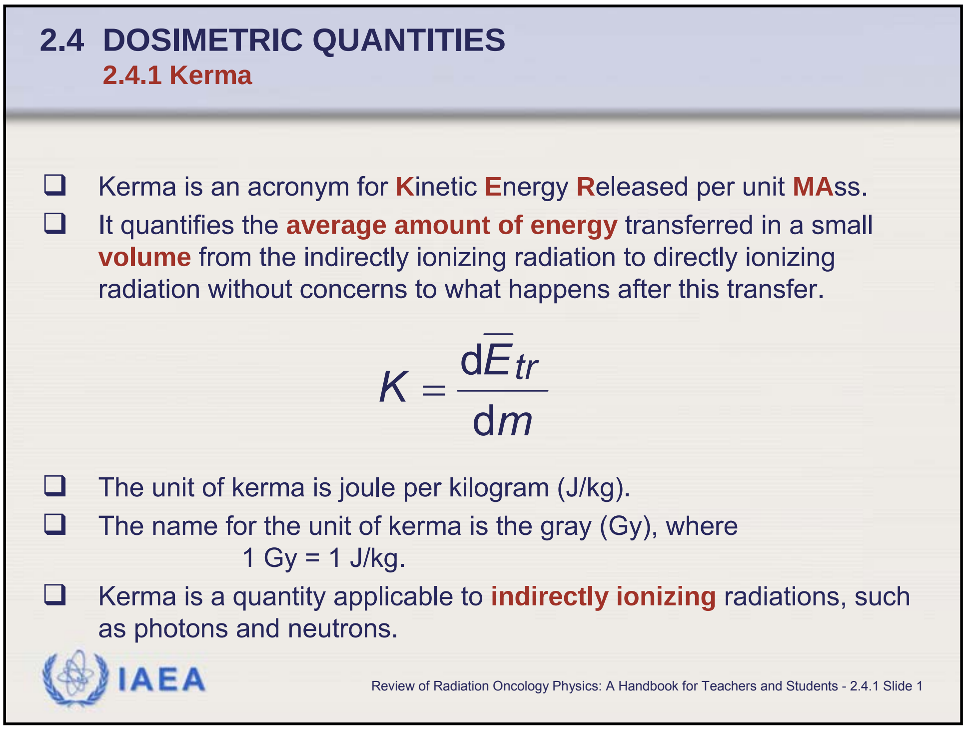
### 2.3.4 Energy Absorption and Energy Transfer

Relation between **"Energy Transfer "** and **"Energy Absorption "**

- ❑ For charged particles, most of the energy loss is directly absorbed → **Energy Absorption**
- ❑ For uncharged particles, energy is transferred in a first step to (secondary) charged particles → **Energy Transfer**.

  In a second step, the secondary charged particles loose their energy according to the general behavior of charged particles (again **Energy Absorption).**

**The energy of uncharged particles like photons or neutrons is imparted to matter in a two stage process.**



## 2.4 DOSIMETRIC QUANTITIES

### 2.4.1 Kerma

- ❑ Kerma is an acronym for **K**inetic **E**nergy **R**eleased per unit **MA**ss.
- ❑ It quantifies the **average amount of energy** transferred in a small **volume** from the indirectly ionizing radiation to directly ionizing radiation without concerns to what happens after this transfer.

$$K = \frac{\mathrm{d}\bar{E}_{tr}}{\mathrm{d}m}$$

- ❑ The unit of kerma is joule per kilogram (J/kg).
- ❑ The name for the unit of kerma is the gray (Gy), where
  1 Gy = 1 J/kg.
- ❑ Kerma is a quantity applicable to **indirectly ionizing** radiations, such as photons and neutrons.

# 2.4 DOSIMETRIC QUANTITIES
## 2.4.1 Kerma

- ❑ The energy transferred to electrons by photons can be expended in two distinct ways:
  - through collision interactions
    (soft collisions and hard collisions);
  - through radiative interactions
    (bremsstrahlung and electron–positron annihilation).
- ❑ The total kerma is therefore usually divided into two components:
  - the collision kerma $K_{col}$
  - the radiative kerma $K_{rad}$.

$$K = K_{col} + K_{rad}$$

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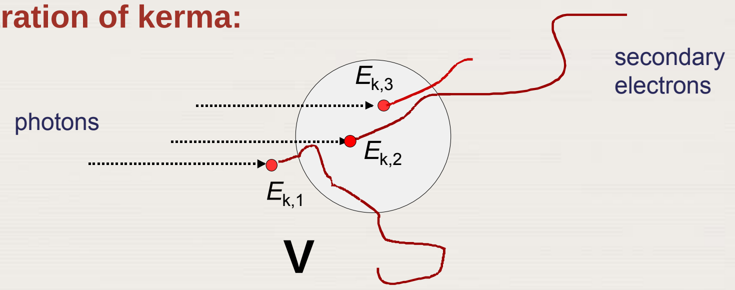
# 2.4 DOSIMETRIC QUANTITIES
## 2.4.1 Kerma

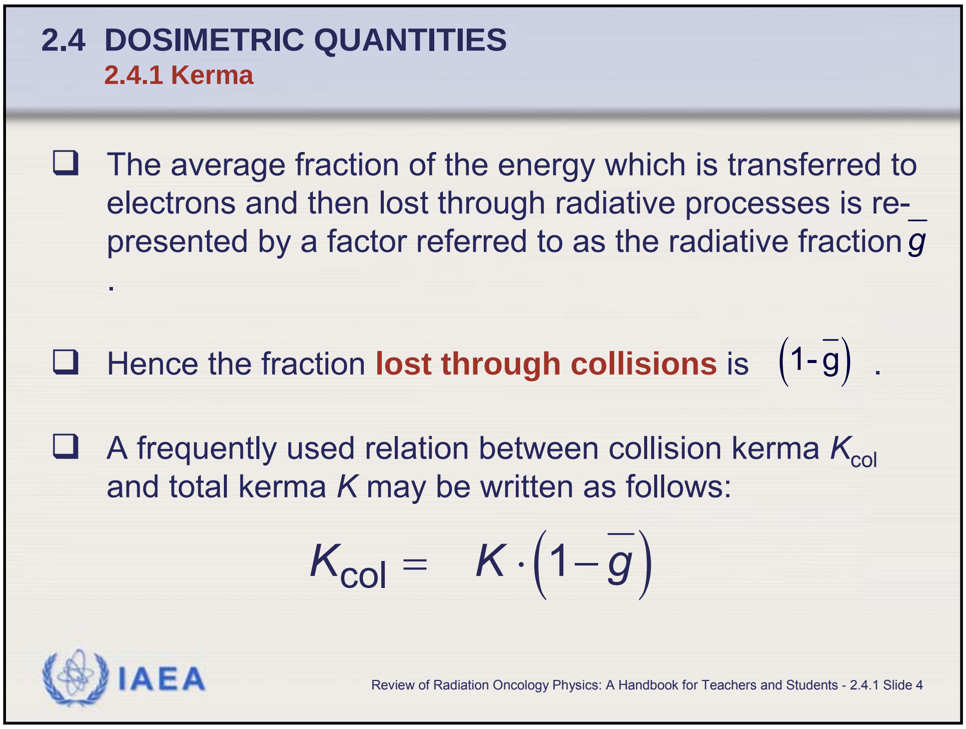
**Illustration of kerma:**



Collision energy transferred in the volume: $E_{tr} = E_{k,2} + E_{k,3}$

where $E_k$ is the initial kinetic energy of the secondary electrons.

Note: $E_{k,1}$ is transferred **outside the volume** and is therefore not taken into account in the definition of kerma!



## 2.4 DOSIMETRIC QUANTITIES

### 2.4.1 Kerma

- The average fraction of the energy which is transferred to electrons and then lost through radiative processes is represented by a factor referred to as the radiative fraction $\bar{g}$ .
- Hence the fraction **lost through collisions** is $(1-\bar{g})$ .
- A frequently used relation between collision kerma $K_{\text{col}}$ and total kerma $K$ may be written as follows:

$$K_{\text{col}} = K \cdot (1-\bar{g})$$

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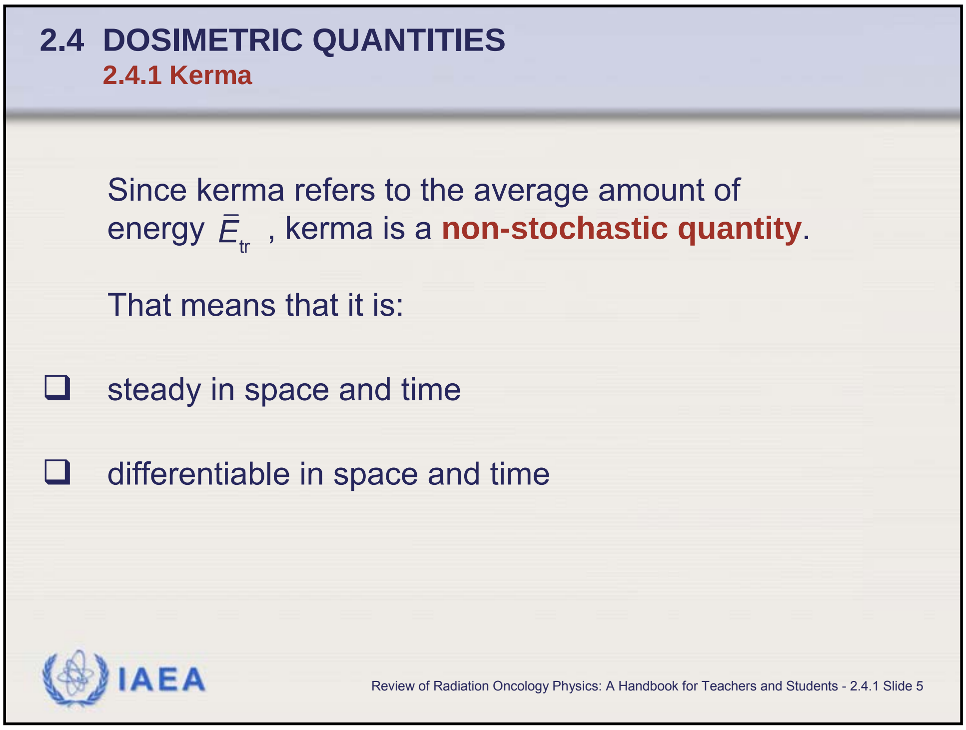

## 2.4 DOSIMETRIC QUANTITIES

### 2.4.1 Kerma

Since kerma refers to the average amount of energy $\bar{E}_{\text{tr}}$ , kerma is a **non-stochastic quantity**.

That means that it is:

- steady in space and time
- differentiable in space and time

## 2.4 DOSIMETRIC QUANTITIES

### 2.4.2 Cema

- Similar to kerma, cema is an acronym for **C**onverted **E**nergy per unit **MA**ss.
- It quantifies the **average amount of energy** converted in a small volume from directly ionizing radiations such as electrons and protons in collisions with atomic electrons without concerns to what happens after this transfer.

$$C = \frac{\mathrm{d}\bar{E}_{\mathrm{c}}}{\mathrm{d}m}$$

- The unit of cema is joule per kilogram (J/kg).
- The name for the unit of kerma is the gray (Gy).

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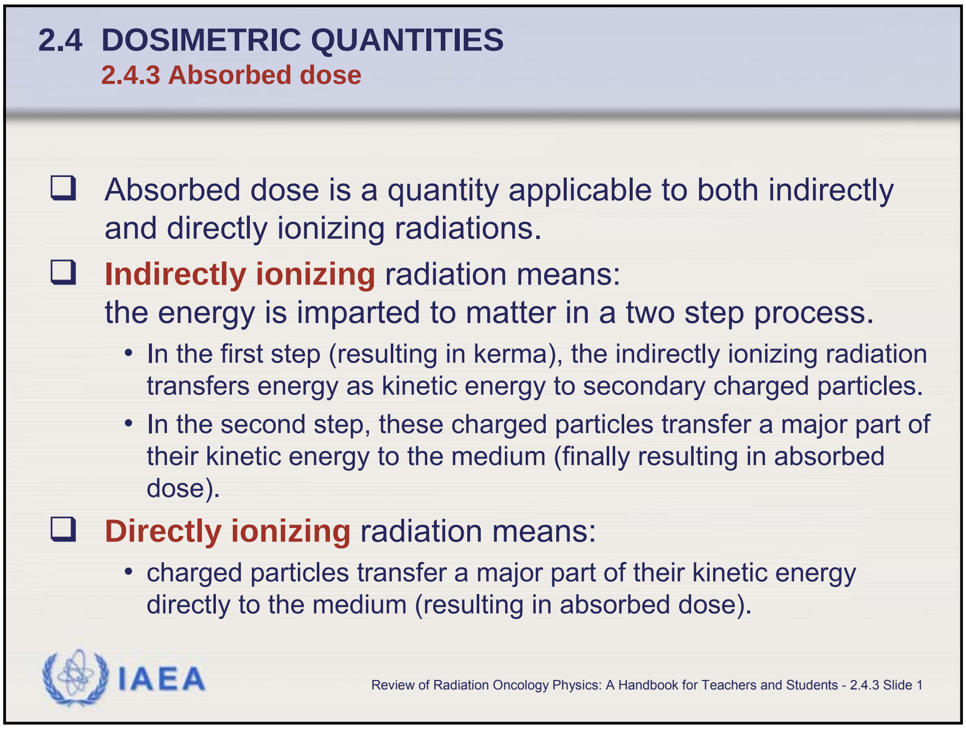
## 2.4 DOSIMETRIC QUANTITIES

### 2.4.2 Cema

**Cema** differs from **kerma** in that:

- Cema involves the energy lost in electronic collisions by the **incoming charged** particles.
- Kerma involves the energy imparted to **outgoing charged** particles.



## 2.4 DOSIMETRIC QUANTITIES

### 2.4.3 Absorbed dose

- ❑ Absorbed dose is a quantity applicable to both indirectly and directly ionizing radiations.
- ❑ **Indirectly ionizing** radiation means:
  the energy is imparted to matter in a two step process.
  - In the first step (resulting in kerma), the indirectly ionizing radiation transfers energy as kinetic energy to secondary charged particles.
  - In the second step, these charged particles transfer a major part of their kinetic energy to the medium (finally resulting in absorbed dose).
- ❑ **Directly ionizing** radiation means:
  - charged particles transfer a major part of their kinetic energy directly to the medium (resulting in absorbed dose).

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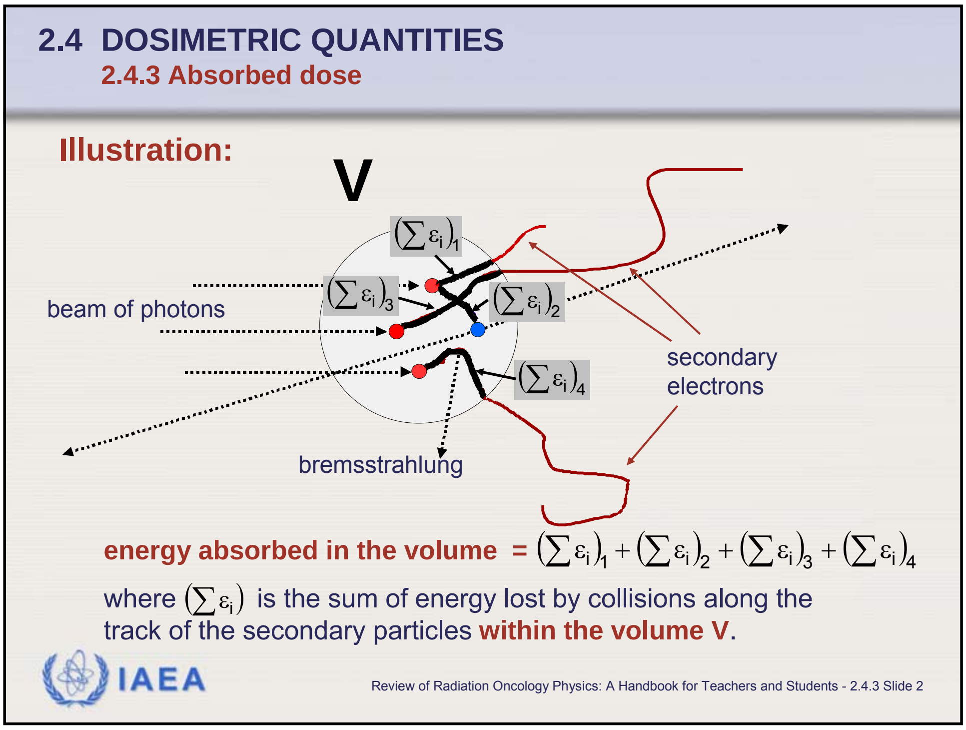

## 2.4 DOSIMETRIC QUANTITIES

### 2.4.3 Absorbed dose

**Illustration:**

**energy absorbed in the volume** $= (\sum \varepsilon_i)_1 + (\sum \varepsilon_i)_2 + (\sum \varepsilon_i)_3 + (\sum \varepsilon_i)_4$

where $(\sum \varepsilon_i)$ is the sum of energy lost by collisions along the track of the secondary particles **within the volume V**.

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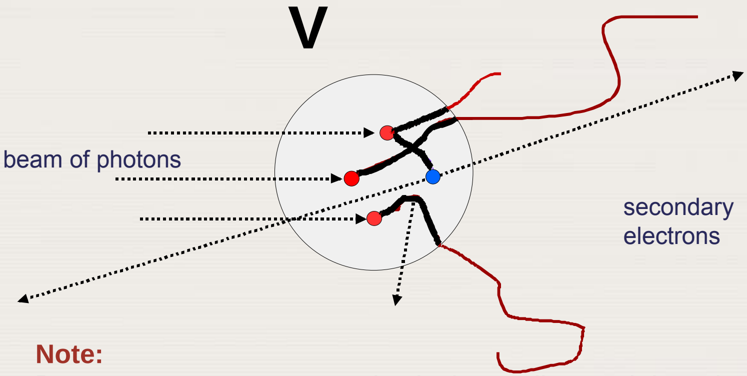
## 2.4 DOSIMETRIC QUANTITIES
### 2.4.3 Absorbed dose



**Note:**
Because electrons are traveling in the medium and deposit energy along their tracks, the absorption of energy (= ▬ ) does not take place at the same location as the transfer of energy described by kerma (= ● ).

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## 2.4 DOSIMETRIC QUANTITIES
### 2.4.3 Absorbed dose

- ☐ As kerma and cema, the absorbed dose is a non stochastic quantity.
- ☐ Absorbed dose, *D*, is related to the stochastic quantity energy imparted $\varepsilon$ by:

$$D = \frac{\mathrm{d}\bar{\varepsilon}}{\mathrm{d}m}$$

- ☐ The unit of absorbed dose is joule per kilogram (J/kg).
- ☐ The name for the unit of absorbed dose is the gray (Gy).

## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

- ❑ Since dosimetric quantities can also be defined as

$$\text{dosimetric quantity} = \text{radiation field quantity} \times \text{mass interaction coefficient} \left[\frac{\text{J}}{\text{kg}}\right]$$

this characteristics needs an inspection into the interaction coefficients of radiation.

- ❑ The following slides refer to electrons and photons. They include some repetitions taken from chapter 1.

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## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.1 Electron interactions

From chapter 1 we know:

- ❑ As an energetic electron traverses matter, it undergoes Coulomb interactions with absorber atoms, i.e., with:
  - Atomic orbital electrons
  - Atomic nuclei

- ❑ Through these collisions the electrons may:
  - Lose their kinetic energy (collision and radiation loss)
  - Change direction of motion (scattering)

## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.1 Electron interactions

- Energy losses are described by **stopping power**.
- Scattering is described by **angular scattering power**.
- Collision between the incident electron and an absorber atom may be:
  - Elastic
  - Inelastic

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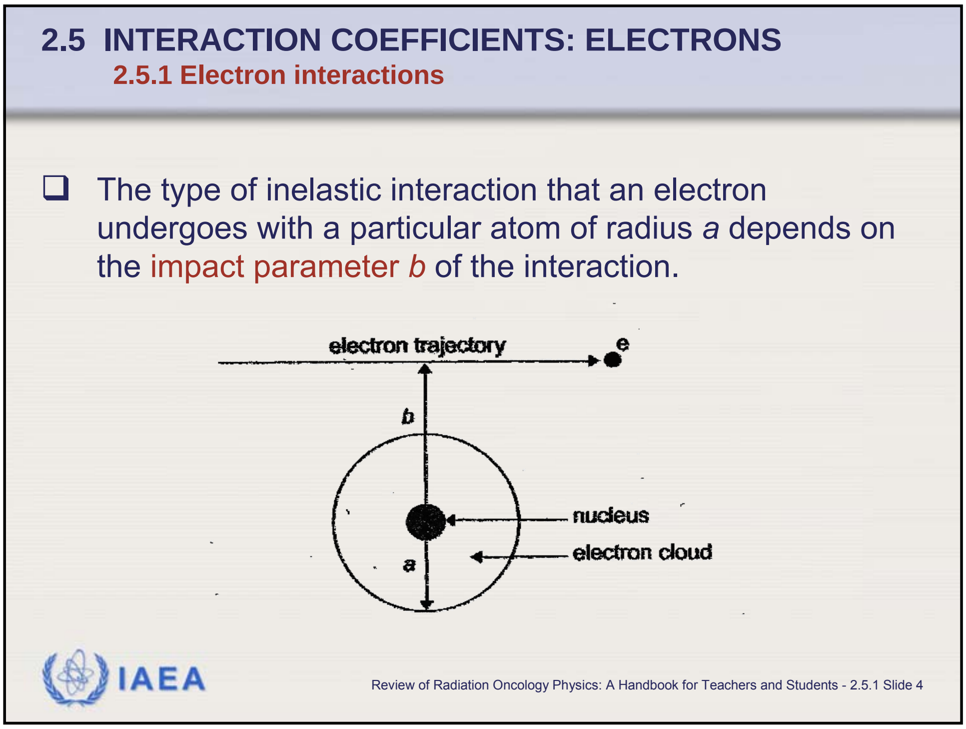
## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.1 Electron interactions

- In an elastic collision the incident electron is deflected from its original path but no energy loss occurs.
- In an inelastic collision with orbital electron the incident electron is deflected from its original path and loses part of its kinetic energy.
- In an inelastic collision with nucleus the incident electron is deflected from its original path and loses part of its kinetic energy in the form of bremsstrahlung.



## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.1 Electron interactions

- ❑ The type of inelastic interaction that an electron undergoes with a particular atom of radius *a* depends on the impact parameter *b* of the interaction.

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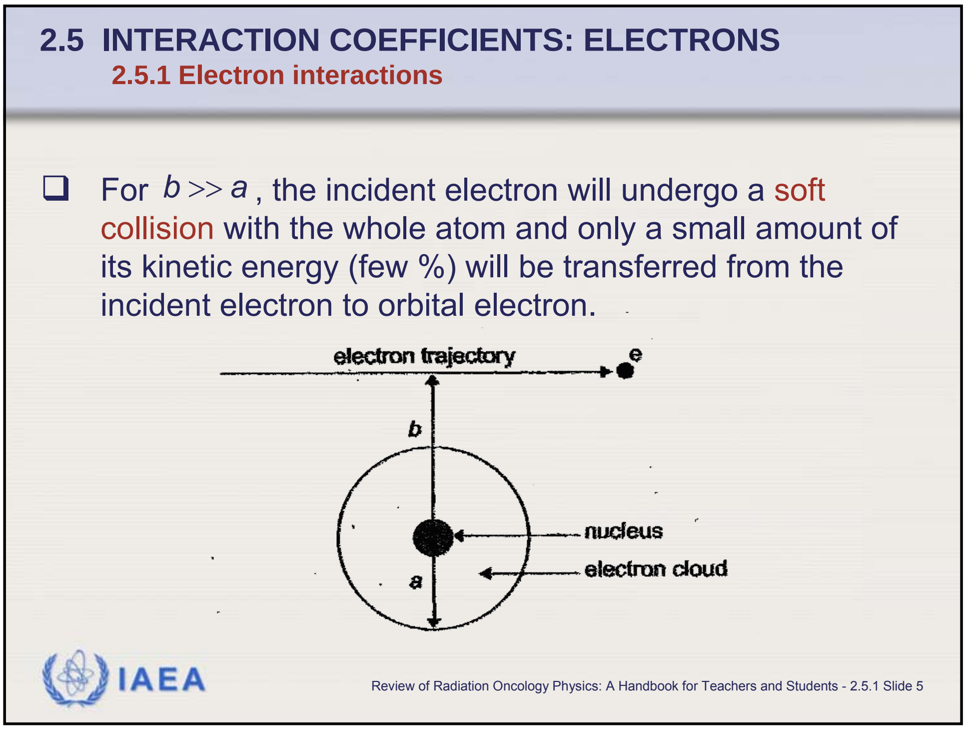

## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.1 Electron interactions

- ❑ For $b >> a$, the incident electron will undergo a soft collision with the whole atom and only a small amount of its kinetic energy (few %) will be transferred from the incident electron to orbital electron.

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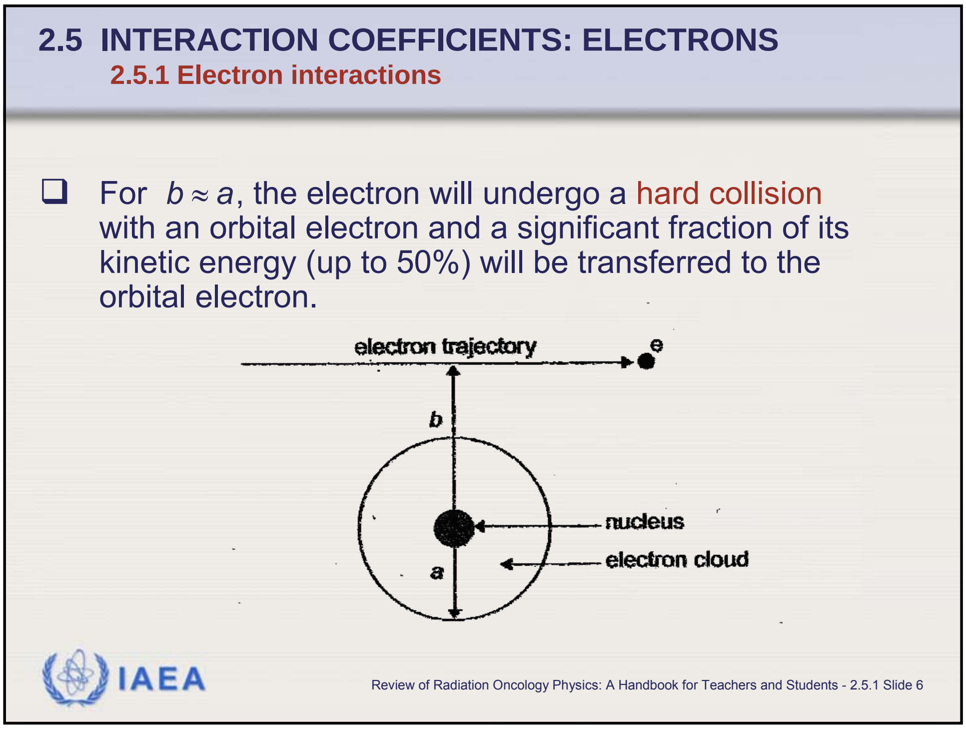

## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.1 Electron interactions

- For $b \approx a$, the electron will undergo a hard collision with an orbital electron and a significant fraction of its kinetic energy (up to 50%) will be transferred to the orbital electron.

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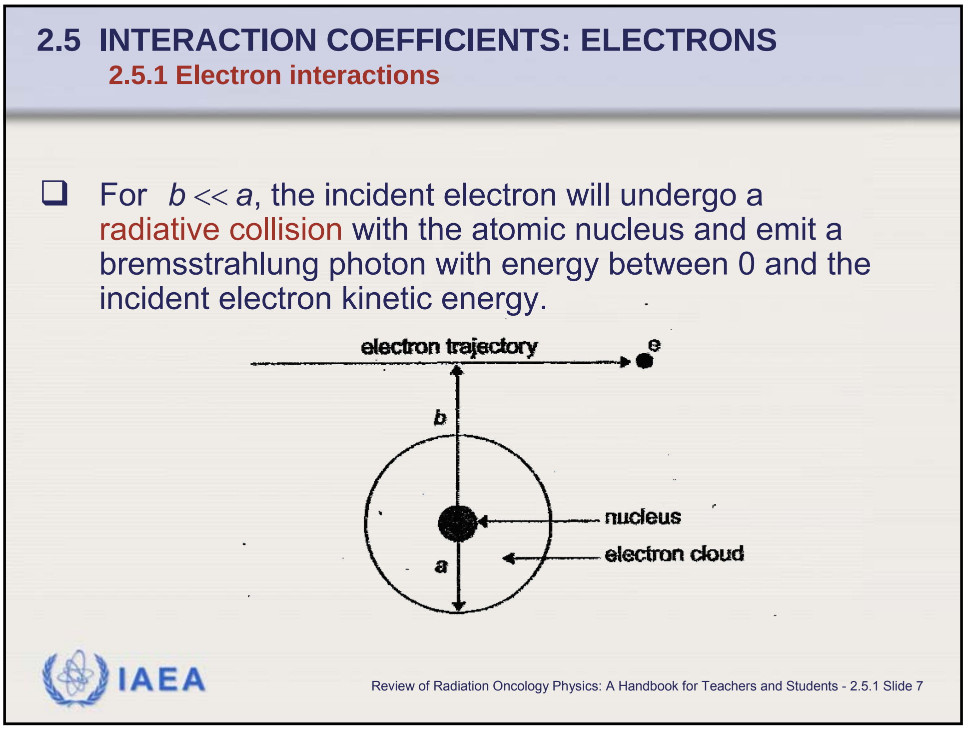

## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.1 Electron interactions

- For $b << a$, the incident electron will undergo a radiative collision with the atomic nucleus and emit a bremsstrahlung photon with energy between 0 and the incident electron kinetic energy.

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## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.1 Electron interactions

- Inelastic collisions between the incident electron and an orbital electron are Coulomb interactions that result in:
  - Atomic ionization:
    Ejection of the orbital electron from the absorber atom.
  - Atomic excitation:
    Transfer of an atomic orbital electron from one allowed orbit (shell) to a higher level allowed orbit.
- Atomic ionizations and excitations result in collision energy losses experienced by the incident electron and are characterized by collision (ionization) stopping power.

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## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.2 Electrons: Stopping power for charged particles

- The total energy loss by incident charged particles through inelastic collisions is described by the

  total linear stopping power $S_{\text{tot}}$

  which represents the average rate of kinetic energy loss $E_K$ by the electron per unit path length $x$:

$$S_{\text{tot}} = \frac{\mathrm{d}E_K}{\mathrm{d}x} \qquad \text{in MeV/cm}$$

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## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.3 Electrons: Mass stopping power

- Division by the density of the absorbing medium almost eliminates the dependence of the mass stopping power on mass density,

- Total mass stopping power $(S/\rho)_{\text{tot}}$ is defined as the linear stopping power divided by the density of the absorbing medium.

$$\left(\frac{S}{\rho}\right)_{\text{tot}} = \frac{1}{\rho}\frac{\mathrm{d}E_{\text{K}}}{\mathrm{d}x} \qquad \text{in MeV cm}^2/\text{g}$$

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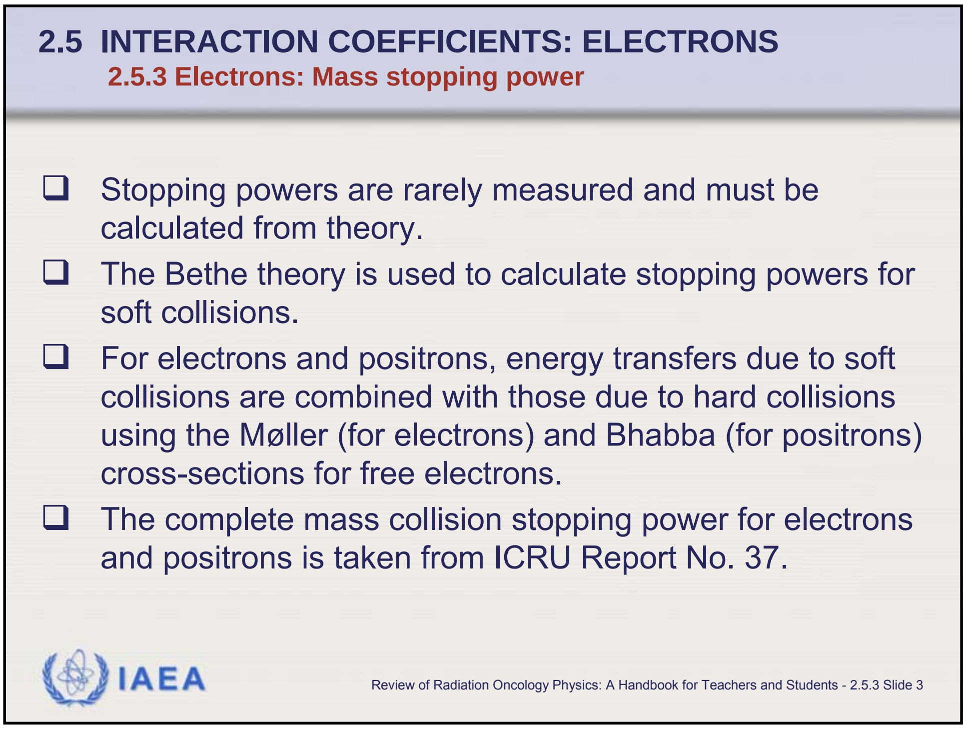
## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.3 Electrons: Mass stopping power

- The total mass stopping power $(S/\rho)_{\text{tot}}$ consists of the two components:
  - **Mass collision stopping power** $(S/\rho)_{\text{col}}$ resulting from electron-orbital electron interactions (atomic ionizations and atomic excitations)
  - **Mass radiation stopping power** $(S/\rho)_{\text{rad}}$ resulting mainly from electron-nucleus interactions (bremsstrahlung production)

$$\left(\frac{S}{\rho}\right)_{tot} = \left(\frac{S}{\rho}\right)_{col} + \left(\frac{S}{\rho}\right)_{rad}$$



## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.3 Electrons: Mass stopping power

- ❑ Stopping powers are rarely measured and must be calculated from theory.
- ❑ The Bethe theory is used to calculate stopping powers for soft collisions.
- ❑ For electrons and positrons, energy transfers due to soft collisions are combined with those due to hard collisions using the Møller (for electrons) and Bhabba (for positrons) cross-sections for free electrons.
- ❑ The complete mass collision stopping power for electrons and positrons is taken from ICRU Report No. 37.

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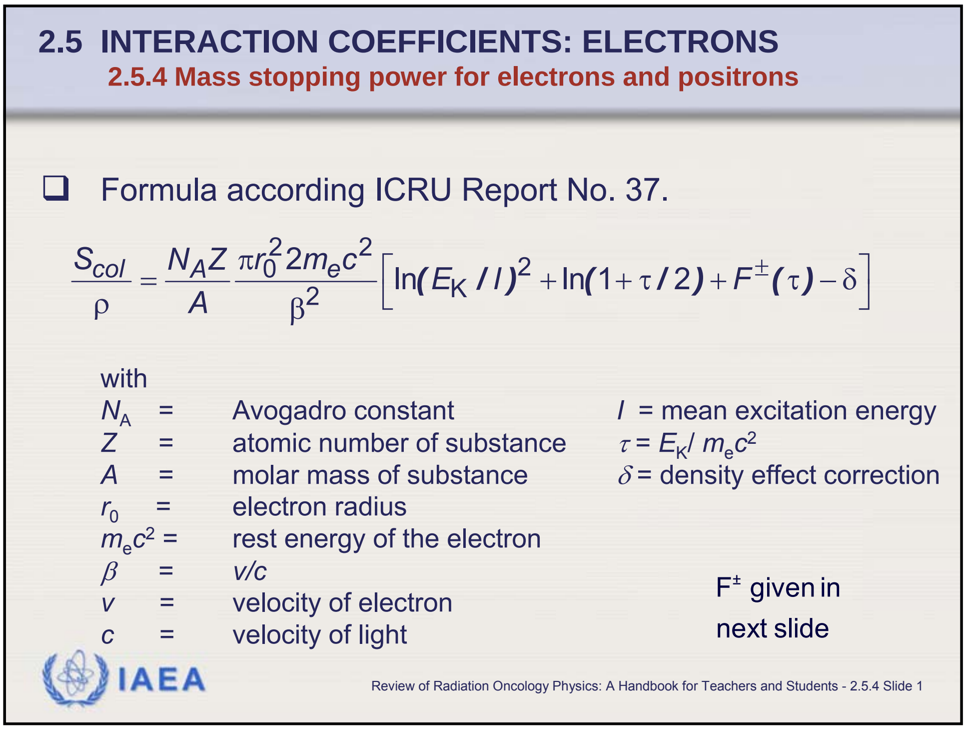

## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.4 Mass stopping power for electrons and positrons

- ❑ Formula according ICRU Report No. 37.

$$\frac{S_{col}}{\rho} = \frac{N_A Z}{A}\,\frac{\pi r_0^2\, 2 m_e c^2}{\beta^2}\left[\ln(E_K/I)^2 + \ln(1+\tau/2) + F^{\pm}(\tau) - \delta\right]$$

with

$N_A$ = Avogadro constant
$Z$ = atomic number of substance
$A$ = molar mass of substance
$r_0$ = electron radius
$m_e c^2$ = rest energy of the electron
$\beta$ = $v/c$
$v$ = velocity of electron
$c$ = velocity of light

$I$ = mean excitation energy
$\tau = E_K / m_e c^2$
$\delta$ = density effect correction

$F^{\pm}$ given in next slide

## 2.5 INTERACTION COEFFICIENTS: ELECTRONS
### 2.5.4 Mass stopping power for electrons and positrons

$F^-$ for electrons is given as:

$$F^- = (1-\beta^2)\left[1+\tau^2/8-(2\tau+1)\ln 2\right]$$

$F^+$ for positrons is given as:

$$F^+ = 2\ln 2-(\beta^2/12)\left[23+14/(\tau+2)+10/(\tau+2)^2+4/(\tau+2)^3\right]$$

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## 2.5 INTERACTION COEFFICIENTS: ELECTRONS
### 2.5.4 Mass stopping power for electrons and positrons

- ☐ The mean **excitation potential *I*** is a geometric mean value of all ionization and excitation potentials of an atom of the absorbing material.
- ☐ *I* values are usually derived from measurements of stopping powers in heavy charged particle beams, for which the effects of scattering in these measurements is minimal.
- ☐ For elemental materials *I* varies approximately linearly with *Z*, with, on average, $I = 11.5 \times Z$.
- ☐ For compounds, *I* is calculated assuming additivity of the collision stopping power, taking into account the fraction by weight of each atom constituent in the compound.

# 2.5 INTERACTION COEFFICIENTS: ELECTRONS
## 2.5.4 Mass stopping power for electrons and positrons

- ❑ Selected data on the mean excitation potential *I as given* in ICRU Report No. 37

| substance | excitation potential in eV |
|---|---|
| hydrogen (molecular gas) | 19.2 |
| carbon (atomic gas) | 62.0 |
| nitrogen (molecular gas) | 82.0 |
| oxygen (molecular gas) | 95.0 |
| air | 85.7 |
| water, liquid | 75.0 |

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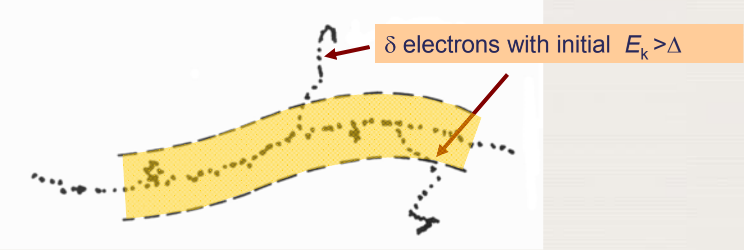
# 2.5 INTERACTION COEFFICIENTS: ELECTRONS
## 2.5.5 Concept of restricted stopping power

**Track of an electron:**



$\delta$ electrons with initial $E_k > \Delta$

Generate a tube around the track such that the radius of the tube includes the start energy of $\delta$ electrons up to a maximum energy $\Delta$.

**$\delta$ electrons with a start energy $E_k > \Delta$ are excluded**.

## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.5 Mass stopping power for electrons and positrons

Definition of the restricted stopping power for charged particles:

- The restricted linear collision stopping power $L_\Delta$ of a material is the quotient of d$E_\Delta$ by d$l$, where d$E_\Delta$ is the energy lost by a charged particle due to soft and hard collisions in traversing a distance d$l$ **minus** the total kinetic energy of the charged particles released with kinetic energies in excess of $\Delta$:

$$L_\Delta = \frac{\mathrm{d}E_\Delta}{\mathrm{d}l}$$

---

## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.5 Mass stopping power for electrons and positrons

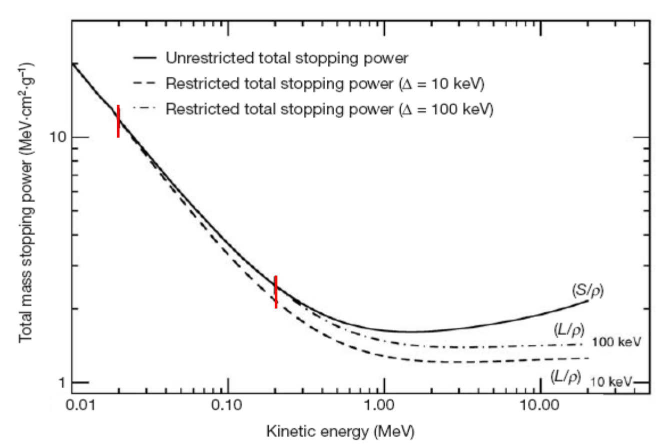
- Note:
  As the threshold for maximum energy transfer in the restricted stopping power increases, the restricted mass stopping power tends to the unrestricted mass stopping power for $\Delta \rightarrow E_K / 2$ .

- Note also that since energy transfers to secondary electrons are limited to $E_K/2$, unrestricted and restricted electron mass stopping powers are identical for kinetic energies lower than or equal to $2\Delta$.

## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.5 Mass stopping power for electrons and positrons

*Unrestricted and restricted (Δ = 10 and 100 keV) total mass stopping powers for carbon (from ICRU Report No. 37)*



Vertical lines indicate the points at which restricted and unrestricted mass stopping powers begin to diverge as the kinetic energy increases.

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## 2.5 INTERACTION COEFFICIENTS: ELECTRONS

### 2.5.5 Mass stopping power for electrons and positrons

The concept of a restricted stopping power is needed

- ☐ in the **Spencer–Attix cavity theory**
- ☐ in some radiobiological models

## 2.6 INTERACTION COEFFICIENTS: PHOTONS

- The energy that is transferred in an photon interaction to a light charged particle (mostly a secondary electron) is called an energy transfer.
- This process is described by the **energy transfer coefficient**

$$\mu_{tr} = \mu \frac{\bar{E}_{tr}}{h\nu}$$

with $\bar{E}_{tr}$ the average energy transferred from the primary photon with energy $h\nu$ to kinetic energy of charged particles ($e^-$ and $e^+$).

## 2.6 INTERACTION COEFFICIENTS: PHOTONS

Repetition:

- A small part of the energy that is transferred in an photon interaction to a light charged particle leads to the
  - production of radiative photons as the secondary charged particles slow down and interact in the medium.
  - These interactions most prominently are bremsstrahlung as a result of Coulomb field interactions between the charged particle and the atomic nuclei.
- This lost through radiative processes is represented by the factor $\bar{g}$ referred to as the radiative fraction.
- The remaining energy is absorbed. This process is described by the **energy absorption coefficient $\mu_{en}$ (or $\mu_{ab}$ )**

$$\mu_{en} = \mu_{tr} \cdot \left(1 - \bar{g}\right)$$

## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.1 Energy fluence and kerma (photons)

- For monoenergetic photons, the total kerma $K$ at a point in a medium:

$$K = \frac{\mathrm{d}\bar{E}_{tr}}{\mathrm{d}m}$$

is related to the energy fluence $\Psi$ at that point in the medium by:

$$K = \Psi \cdot \frac{\mu_{\mathrm{tr}}}{\rho}$$

where $(\mu_{\mathrm{tr}}/\rho)$ is the mass–energy transfer coefficient for the monoenergetic photons in the medium.

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## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.1 Energy fluence and kerma (photons)

- For monoenergetic photons the collision kerma $K_{\mathrm{col}}$ at a point in a medium:

$$K_{\mathrm{col}} = K \cdot \left(1 - \bar{g}\right)$$

is related to the energy fluence $\Psi$ at that point in the medium by:

$$K_{\mathrm{col}} = \Psi \cdot \frac{\mu_{\mathrm{en}}}{\rho}$$

where $(\mu_{\mathrm{en}}/\rho)$ is the mass–energy absorption coefficient for monoenergetic photons in the medium.

## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.1 Energy fluence and kerma (photons)

- For **polyenergetic** beams a similar relation exists.
- If a photon energy fluence spectrum (that is the energy fluence differential in energy), $\Psi_E(E)$ is present at the point of interest, the collision kerma $K_{\text{col}}$ at that point is obtained by:

$$K_{\text{col}} = \int_0^{E_{\max}} \Psi_E(E) \cdot \left( \frac{\mu_{\text{en}}}{\rho} \right) \mathrm{d}E$$

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## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.1 Energy fluence and kerma (photons)

- One may use the following shorthand notation for the **mean mass–energy absorption coefficient.**
- That is, the mass–energy absorption coefficient is **averaged** over the energy fluence spectrum:

$$\left( \frac{\bar{\mu}_{\text{en}}}{\rho} \right) = \frac{\displaystyle\int_0^{E_{\max}} \Psi_E(E) \cdot \left( \frac{\mu_{\text{en}}(E)}{\rho} \right) \mathrm{d}E}{\displaystyle\int_0^{E_{\max}} \Psi_E(E)\, \mathrm{d}E}$$

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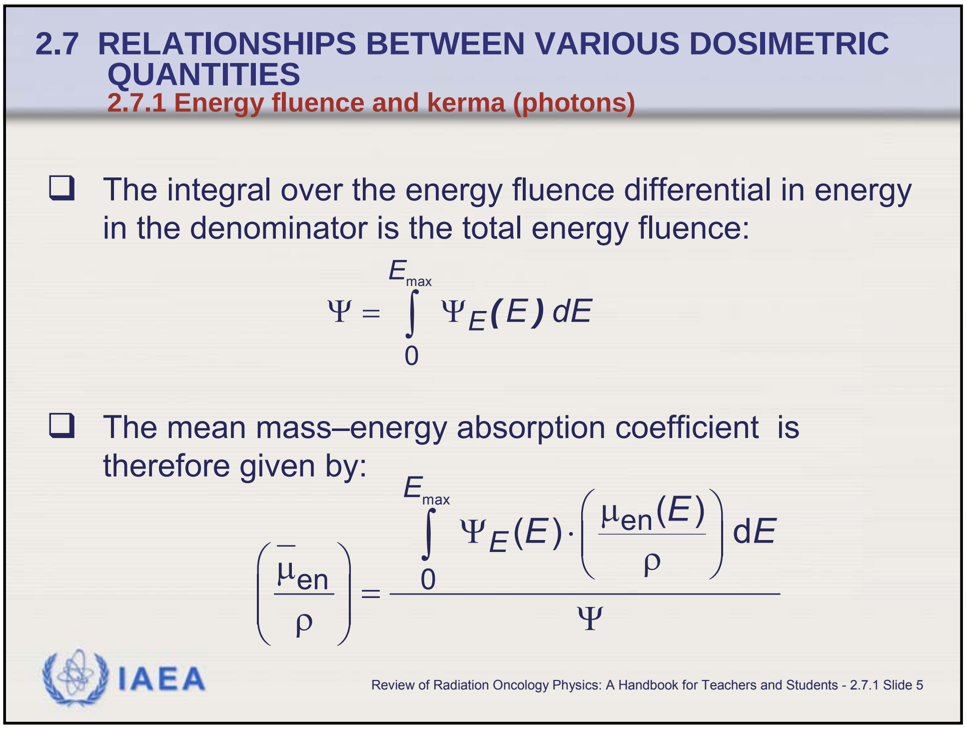

## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.1 Energy fluence and kerma (photons)

- ❑ The integral over the energy fluence differential in energy in the denominator is the total energy fluence:

$$\Psi = \int_0^{E_{\max}} \Psi_E(E)\, dE$$

- ❑ The mean mass–energy absorption coefficient is therefore given by:

$$\left(\frac{\bar{\mu}_{\mathrm{en}}}{\rho}\right) = \frac{\int_0^{E_{\max}} \Psi_E(E) \cdot \left(\frac{\mu_{\mathrm{en}}(E)}{\rho}\right) \mathrm{d}E}{\Psi}$$

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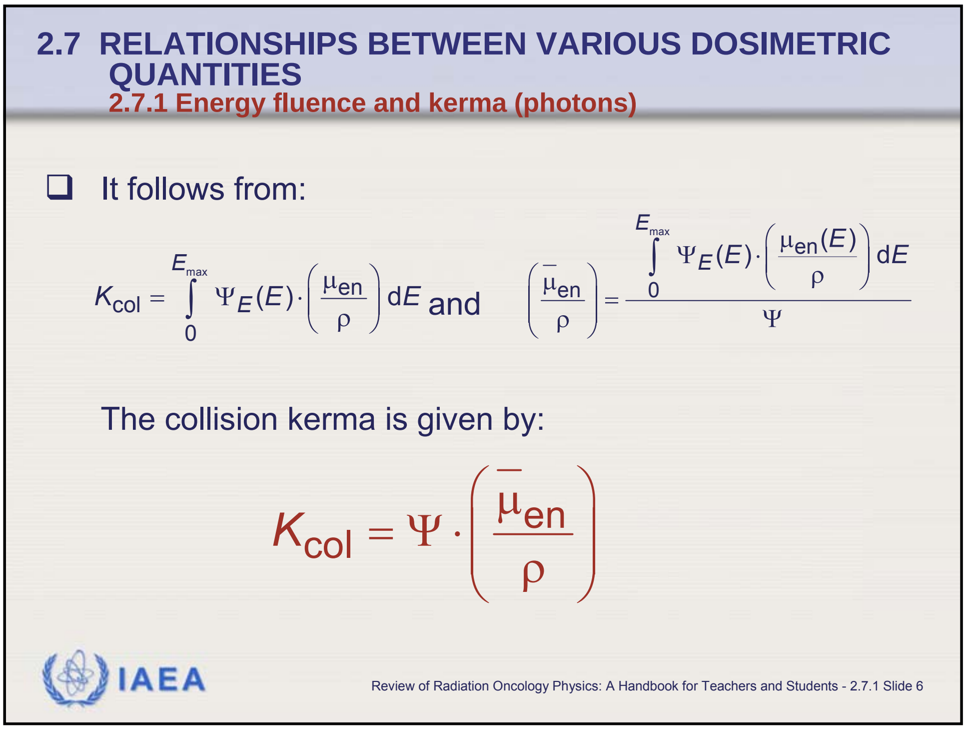

## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.1 Energy fluence and kerma (photons)

- ❑ It follows from:

$$K_{\mathrm{col}} = \int_0^{E_{\max}} \Psi_E(E) \cdot \left(\frac{\mu_{\mathrm{en}}}{\rho}\right) \mathrm{d}E \quad \text{and} \quad \left(\frac{\bar{\mu}_{\mathrm{en}}}{\rho}\right) = \frac{\int_0^{E_{\max}} \Psi_E(E) \cdot \left(\frac{\mu_{\mathrm{en}}(E)}{\rho}\right) \mathrm{d}E}{\Psi}$$

The collision kerma is given by:

$$K_{\mathrm{col}} = \Psi \cdot \left(\frac{\bar{\mu}_{\mathrm{en}}}{\rho}\right)$$

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## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.1 Energy fluence and kerma (photons)

- If one compares the collision kerma between a medium 1 and a medium 2, **both at the same energy fluence $\Psi$**, one can obtain the frequently used relation:

$$\frac{K_{col,2}}{K_{col,1}} = \frac{\Psi \cdot \left(\frac{\bar{\mu}_{en}}{\rho}\right)_2}{\Psi \cdot \left(\frac{\bar{\mu}_{en}}{\rho}\right)_1} = \left(\frac{\bar{\mu}_{en}}{\rho}\right)_{2,1}$$

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## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.1 Energy fluence and kerma (photons)

- In some cases where the energy fluence is not equal in medium 1 and medium 2, the **fluence ratio $\Psi_{2,1}$** can be assumed to be **unity** through a proper scaling of dimensions (using the scaling theorem):
  - for very similar materials
  - for situations in which the mass of material 2 is sufficient to provide buildup but at the same time small enough so as not to disturb the photon fluence in material 1 (for example for a dose to a small mass of tissue in air)

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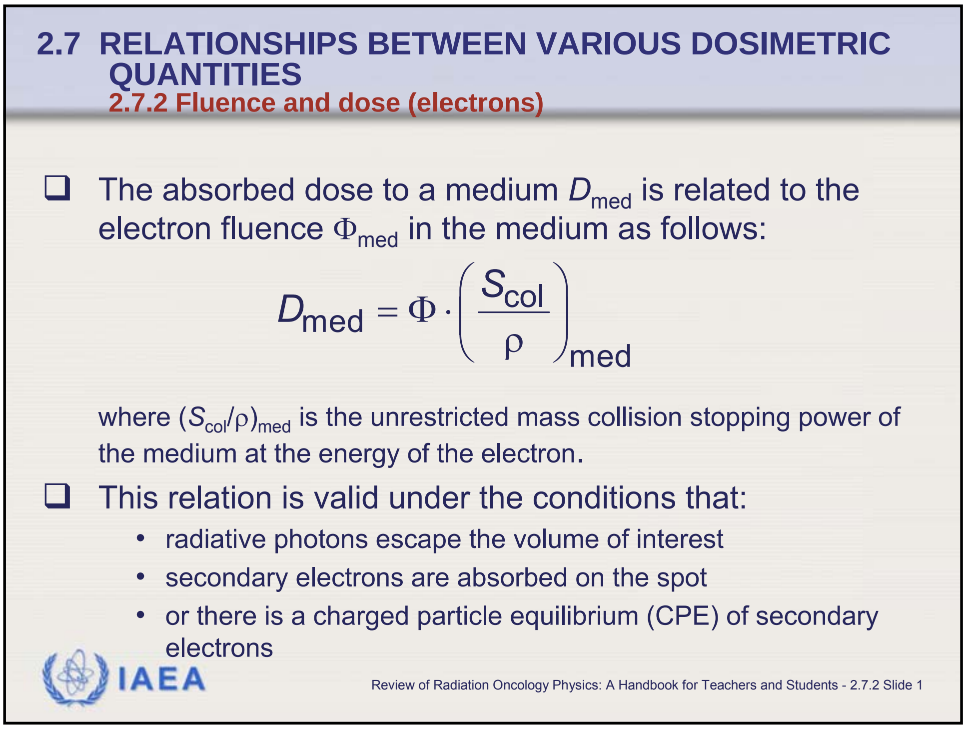


## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

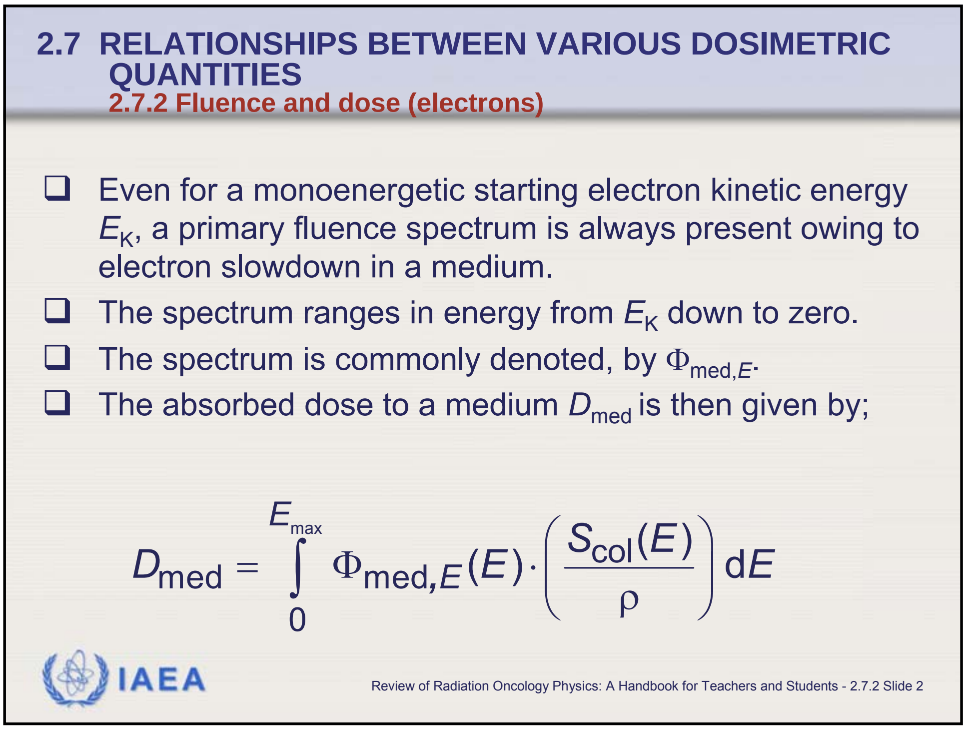
### 2.7.2 Fluence and dose (electrons)

- ☐ The absorbed dose to a medium $D_{\text{med}}$ is related to the electron fluence $\Phi_{\text{med}}$ in the medium as follows:

$$D_{\text{med}} = \Phi \cdot \left( \frac{S_{\text{col}}}{\rho} \right)_{\text{med}}$$

where $(S_{\text{col}}/\rho)_{\text{med}}$ is the unrestricted mass collision stopping power of the medium at the energy of the electron.

- ☐ This relation is valid under the conditions that:
  - radiative photons escape the volume of interest
  - secondary electrons are absorbed on the spot
  - or there is a charged particle equilibrium (CPE) of secondary electrons



## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.2 Fluence and dose (electrons)

- ☐ Even for a monoenergetic starting electron kinetic energy $E_K$, a primary fluence spectrum is always present owing to electron slowdown in a medium.
- ☐ The spectrum ranges in energy from $E_K$ down to zero.
- ☐ The spectrum is commonly denoted, by $\Phi_{\text{med},E}$.
- ☐ The absorbed dose to a medium $D_{\text{med}}$ is then given by;

$$D_{\text{med}} = \int_0^{E_{\max}} \Phi_{\text{med},E}(E) \cdot \left( \frac{S_{\text{col}}(E)}{\rho} \right) dE$$

## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.2 Fluence and dose (electrons)

❑ One may again use a shorthand notation for the collision stopping power **averaged** over the fluence spectrum:

$$\left(\frac{\bar{S}_{col}}{\rho}\right)_{med} = \frac{1}{\Phi_{med}} \int_{0}^{E_{max}} \Phi_{med,E}(E) \cdot \left(\frac{S_{col}(E)}{\rho}\right)_{med} \mathrm{d}E$$

The collision kerma is then given by:

$$D_{med} = \Phi_{med} \cdot \left(\frac{\bar{S}_{col}}{\rho}\right)$$

---

## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

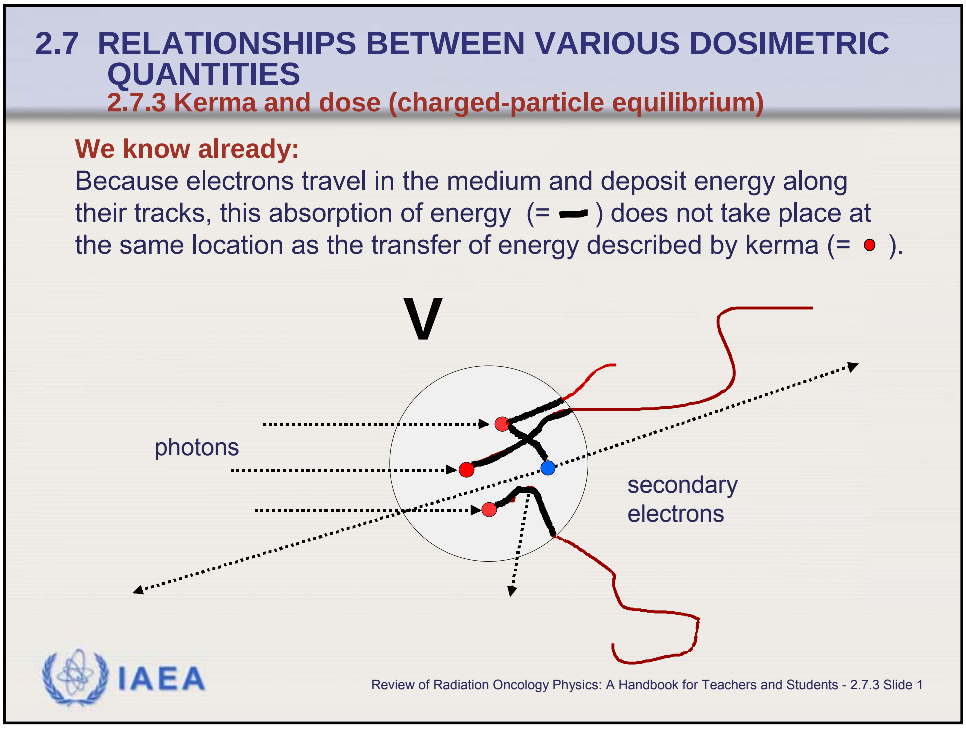
### 2.7.2 Fluence and dose (electrons)

❑ If one compares the absorbed dose between a medium 1 and a medium 2, both at the same fluence:

$$\Phi_{med1} = \Phi_{med2}$$

one can obtain the frequently used relation:

$$\frac{D_{med_2}}{D_{med_1}} = \frac{\Phi_{med_2} \cdot \left(\frac{\bar{S}_{col}}{\rho}\right)_{med_2}}{\Phi_{med_1} \cdot \left(\frac{\bar{S}_{col}}{\rho}\right)_{med_1}} = \left(\frac{\bar{S}_{col}}{\rho}\right)_{med_2,med_1}$$



## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.3 Kerma and dose (charged-particle equilibrium)

**We know already:**
Because electrons travel in the medium and deposit energy along their tracks, this absorption of energy (= ▬ ) does not take place at the same location as the transfer of energy described by kerma (= ● ).

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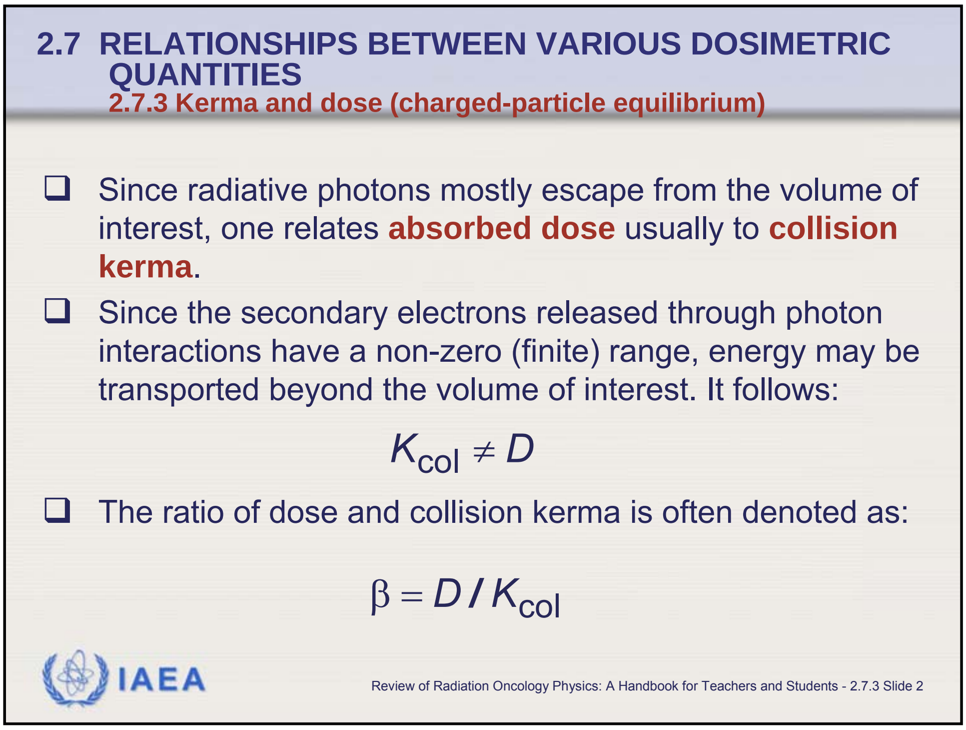

## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.3 Kerma and dose (charged-particle equilibrium)

☐ Since radiative photons mostly escape from the volume of interest, one relates **absorbed dose** usually to **collision kerma**.

☐ Since the secondary electrons released through photon interactions have a non-zero (finite) range, energy may be transported beyond the volume of interest. It follows:

$$K_{\mathrm{col}} \neq D$$

☐ The ratio of dose and collision kerma is often denoted as:

$$\beta = D / K_{\mathrm{col}}$$

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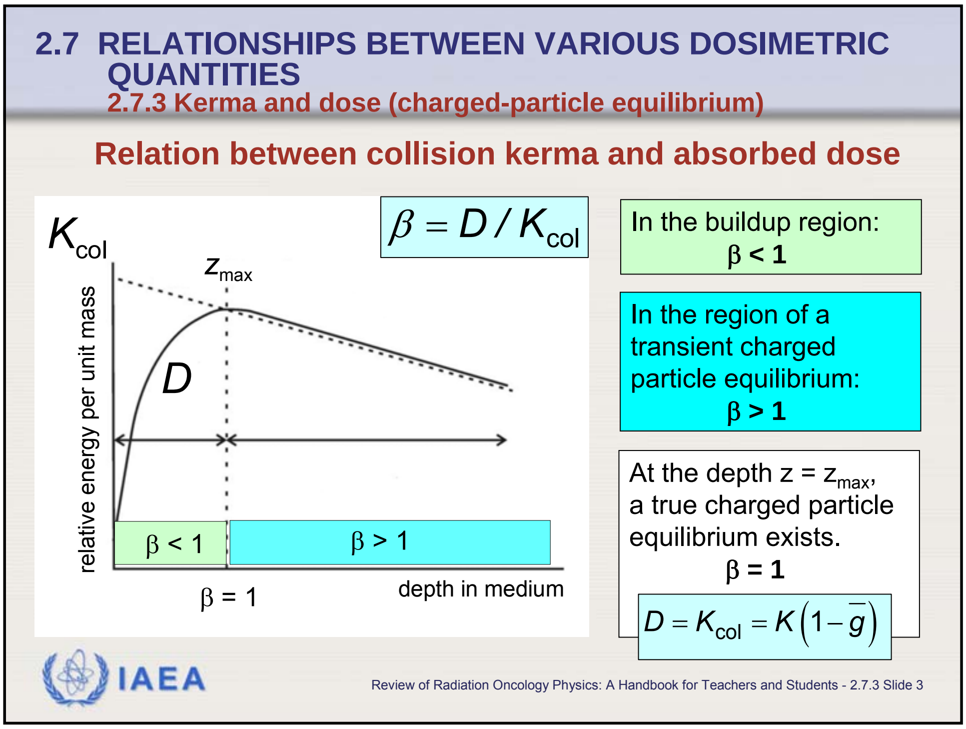


## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.3 Kerma and dose (charged-particle equilibrium)

### Relation between collision kerma and absorbed dose

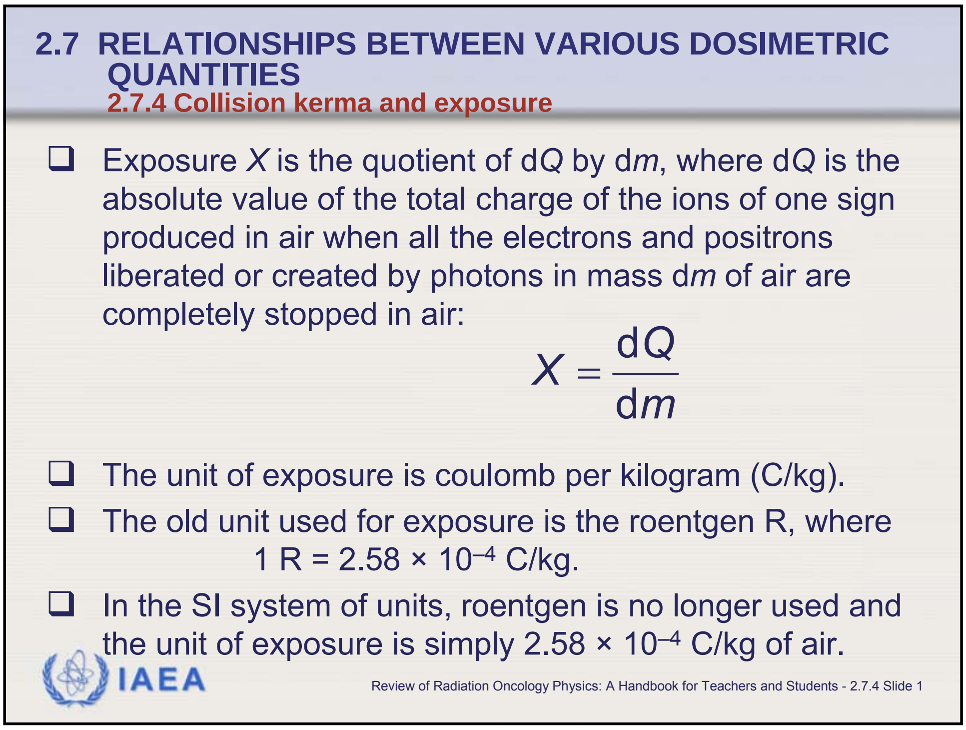
$$\beta = D / K_{\text{col}}$$

In the buildup region:
**β < 1**

In the region of a transient charged particle equilibrium:
**β > 1**

At the depth $z = z_{\max}$, a true charged particle equilibrium exists.
**β = 1**

$$D = K_{\text{col}} = K(1 - \bar{g})$$



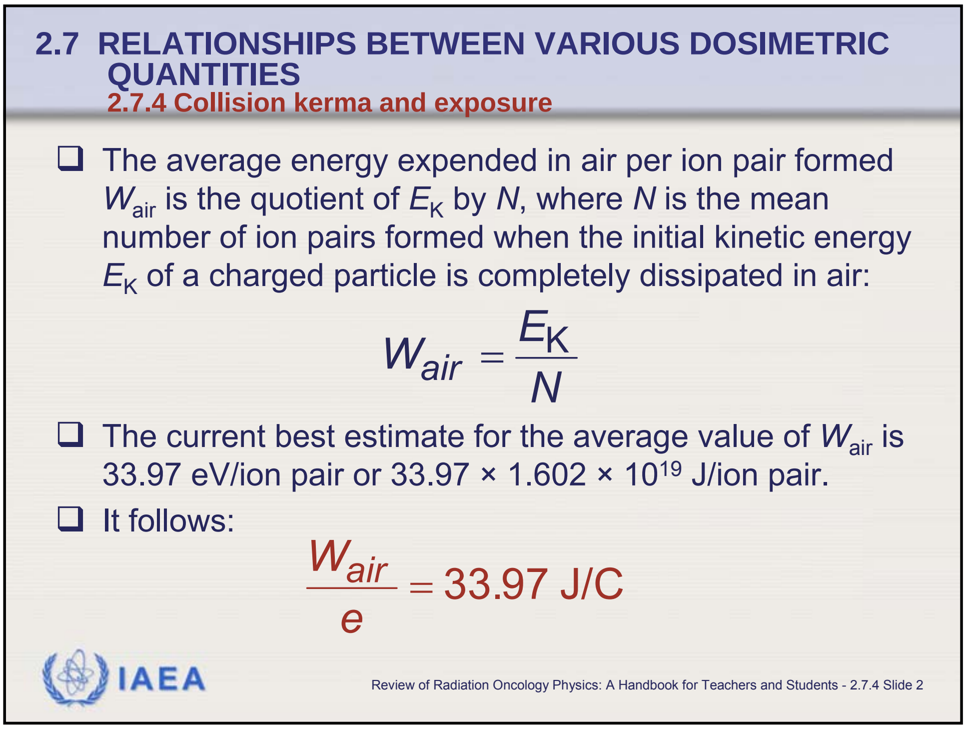
## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.4 Collision kerma and exposure

- ☐ Exposure $X$ is the quotient of d$Q$ by d$m$, where d$Q$ is the absolute value of the total charge of the ions of one sign produced in air when all the electrons and positrons liberated or created by photons in mass d$m$ of air are completely stopped in air:

$$X = \frac{\mathrm{d}Q}{\mathrm{d}m}$$

- ☐ The unit of exposure is coulomb per kilogram (C/kg).
- ☐ The old unit used for exposure is the roentgen R, where
  1 R = 2.58 × $10^{-4}$ C/kg.
- ☐ In the SI system of units, roentgen is no longer used and the unit of exposure is simply 2.58 × $10^{-4}$ C/kg of air.



## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.4 Collision kerma and exposure

- ❑ The average energy expended in air per ion pair formed $W_{\text{air}}$ is the quotient of $E_K$ by $N$, where $N$ is the mean number of ion pairs formed when the initial kinetic energy $E_K$ of a charged particle is completely dissipated in air:

$$W_{air} = \frac{E_K}{N}$$

- ❑ The current best estimate for the average value of $W_{\text{air}}$ is 33.97 eV/ion pair or 33.97 × 1.602 × $10^{19}$ J/ion pair.
- ❑ It follows:

$$\frac{W_{air}}{e} = 33.97 \text{ J/C}$$

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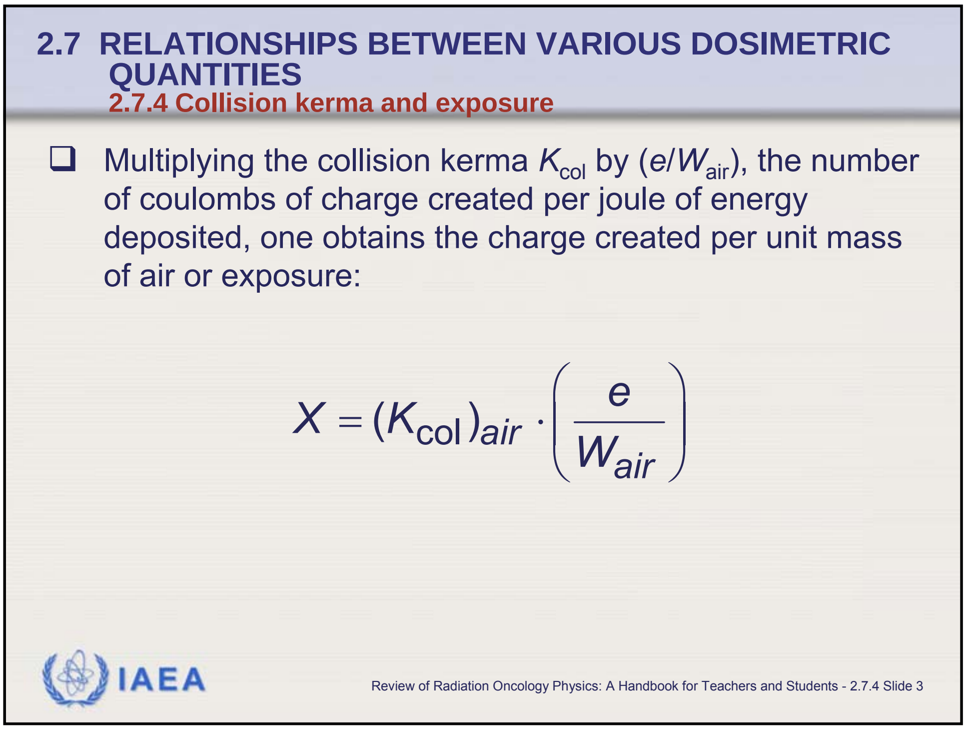

## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.4 Collision kerma and exposure

- ❑ Multiplying the collision kerma $K_{\text{col}}$ by ($e/W_{\text{air}}$), the number of coulombs of charge created per joule of energy deposited, one obtains the charge created per unit mass of air or exposure:

$$X = (K_{\text{col}})_{air} \cdot \left( \frac{e}{W_{air}} \right)$$

## 2.7 RELATIONSHIPS BETWEEN VARIOUS DOSIMETRIC QUANTITIES

### 2.7.4 Collision kerma and exposure

- ❑ Since

$$D = K_{\text{col}}\, K\left(1-\bar{g}\right)$$

it follows:

$$K_{\text{air}} = X \cdot \left(\frac{W_{\text{air}}}{e}\right)\frac{1}{1-\bar{g}}$$

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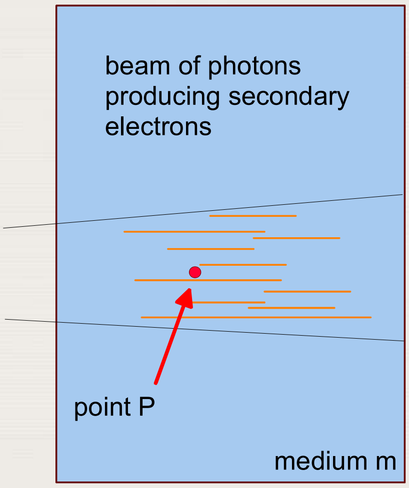
## 2.8 CAVITY THEORY

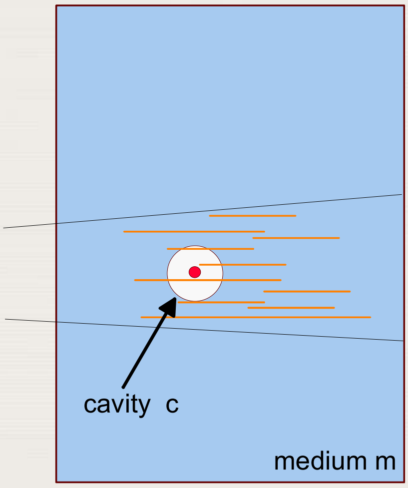
- ❑ Consider a point P within a medium m within a beam of photon radiation (right).
- ❑ The absorbed dose at point P can be calculated by:

$$D_{\text{med}}(\text{P}) = \Phi \cdot \left(\bar{S}/\rho\right)_{\text{med}}$$



## 2.8 CAVITY THEORY

- In order to **measure** the absorbed dose at point P in the medium, it is necessary to introduce a radiation sensitive device (dosimeter) into the medium.
- The sensitive medium of the dosimeter is frequently called a **cavity**.
- Generally, the sensitive medium of the cavity will not be of the same material as the medium in which it is embedded.



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## 2.8 CAVITY THEORY

- The measured absorbed dose $D_{cav}$ within the entire cavity can also be calculated by:

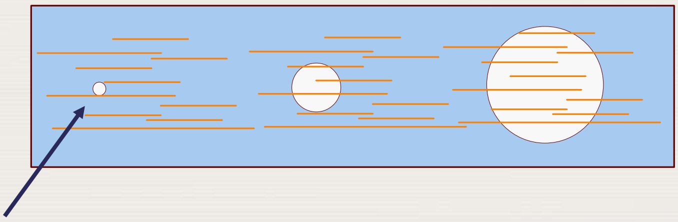
$$D_{cav} = \int_{V_{cav}} \int_{E=0}^{E_{max}} \Phi_{E,\vec{r}}(E,\vec{r}) \frac{S_{cav}(E)}{\rho} \, dE \, d\vec{r}$$

- If the material of the cavity differs in atomic number and density from that of the medium, the measured absorbed dose to the cavity will be different from the absorbed dose to the medium at point P.

$$D_{cav} \neq D_{med}(P)$$

## 2.8 CAVITY THEORY

- ❑ Cavity sizes are referred to as **small, intermediate or large** in comparison with the **ranges of secondary charged particles** produced by photons in the cavity medium.



- ❑ The case where the range of charged particles (electrons) is much larger than the cavity dimensions (i.e. the cavity is regarded as small) is of special interest.

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## 2.8 CAVITY THEORY

- ❑ In order to determine $D_m$ from $D_c$ , various cavity theories have been developed, which depend on the size of the cavity.

Examples are:

- ❑ for small cavities:
  - the **Bragg–Gray theory**
  - and **Spencer–Attix theory**
- ❑ for cavities of intermediate sizes:
  - the **Burlin theory**.

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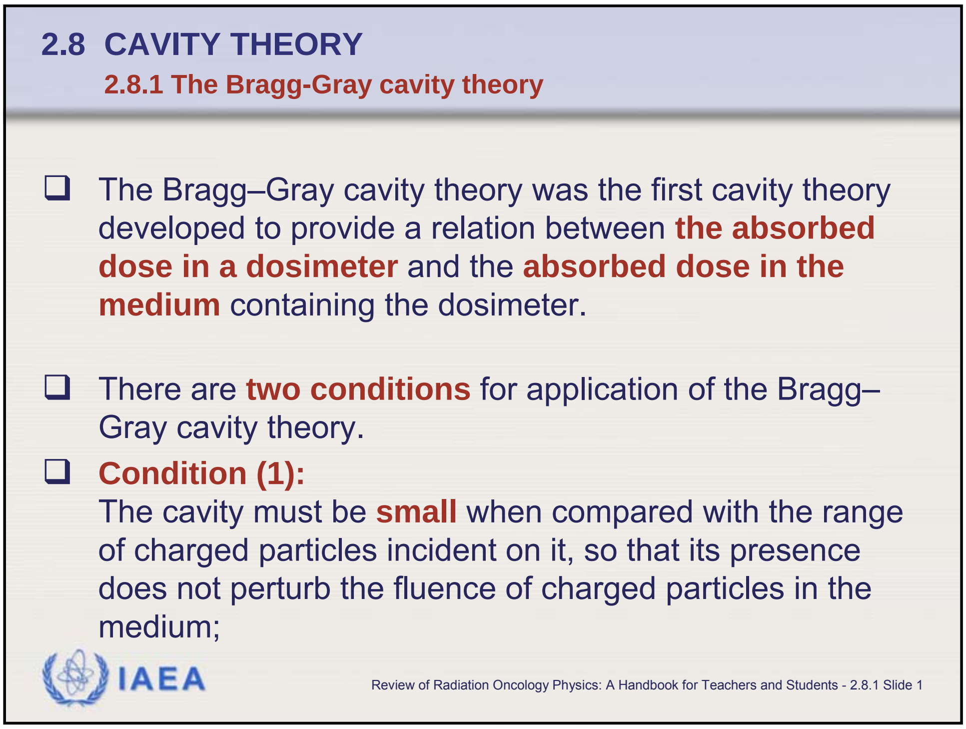
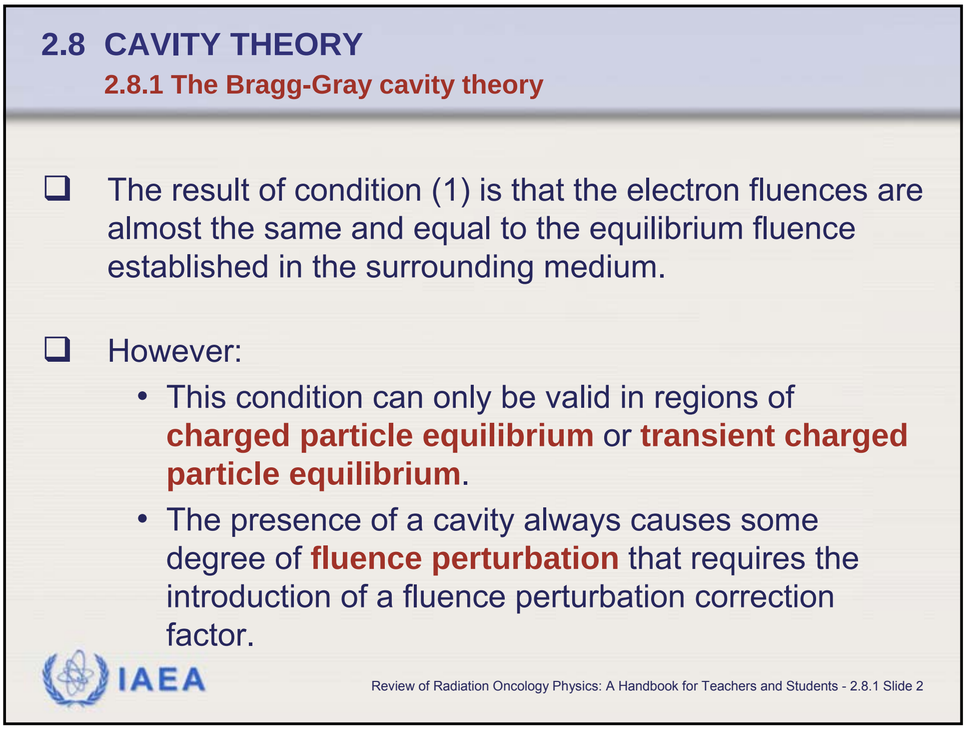


## 2.8 CAVITY THEORY

### 2.8.1 The Bragg-Gray cavity theory

- The Bragg–Gray cavity theory was the first cavity theory developed to provide a relation between **the absorbed dose in a dosimeter** and the **absorbed dose in the medium** containing the dosimeter.

- There are **two conditions** for application of the Bragg–Gray cavity theory.
- **Condition (1):**
  The cavity must be **small** when compared with the range of charged particles incident on it, so that its presence does not perturb the fluence of charged particles in the medium;

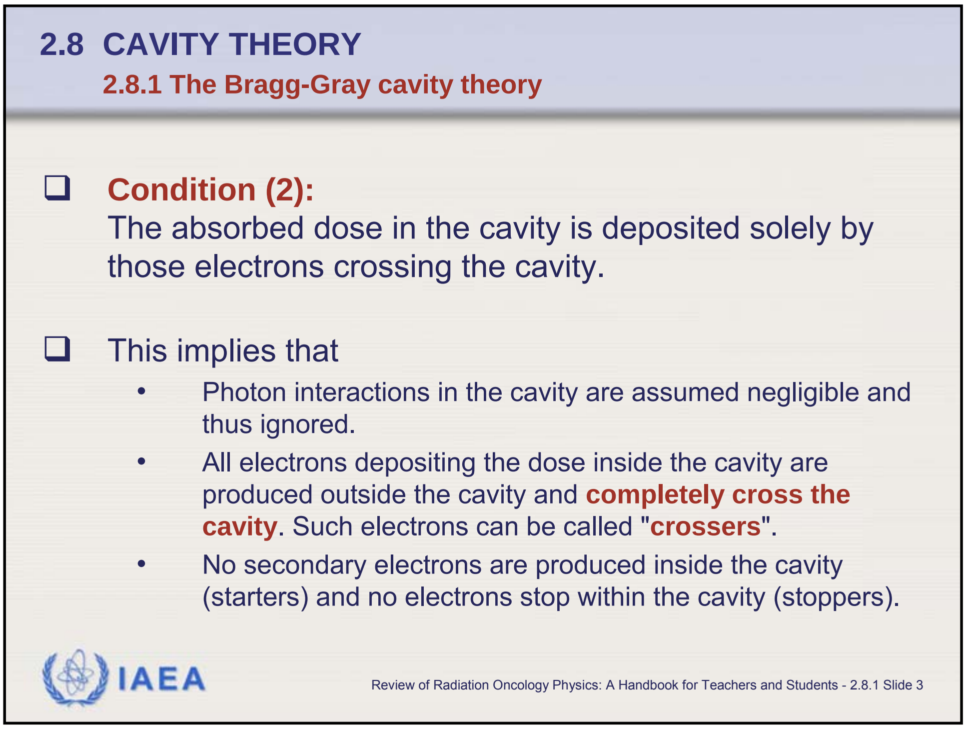


## 2.8 CAVITY THEORY

### 2.8.1 The Bragg-Gray cavity theory

- The result of condition (1) is that the electron fluences are almost the same and equal to the equilibrium fluence established in the surrounding medium.

- However:
  - This condition can only be valid in regions of **charged particle equilibrium** or **transient charged particle equilibrium**.
  - The presence of a cavity always causes some degree of **fluence perturbation** that requires the introduction of a fluence perturbation correction factor.



## 2.8 CAVITY THEORY

### 2.8.1 The Bragg-Gray cavity theory

❑ **Condition (2):**
The absorbed dose in the cavity is deposited solely by those electrons crossing the cavity.

❑ This implies that

- Photon interactions in the cavity are assumed negligible and thus ignored.
- All electrons depositing the dose inside the cavity are produced outside the cavity and **completely cross the cavity**. Such electrons can be called "**crossers**".
- No secondary electrons are produced inside the cavity (starters) and no electrons stop within the cavity (stoppers).

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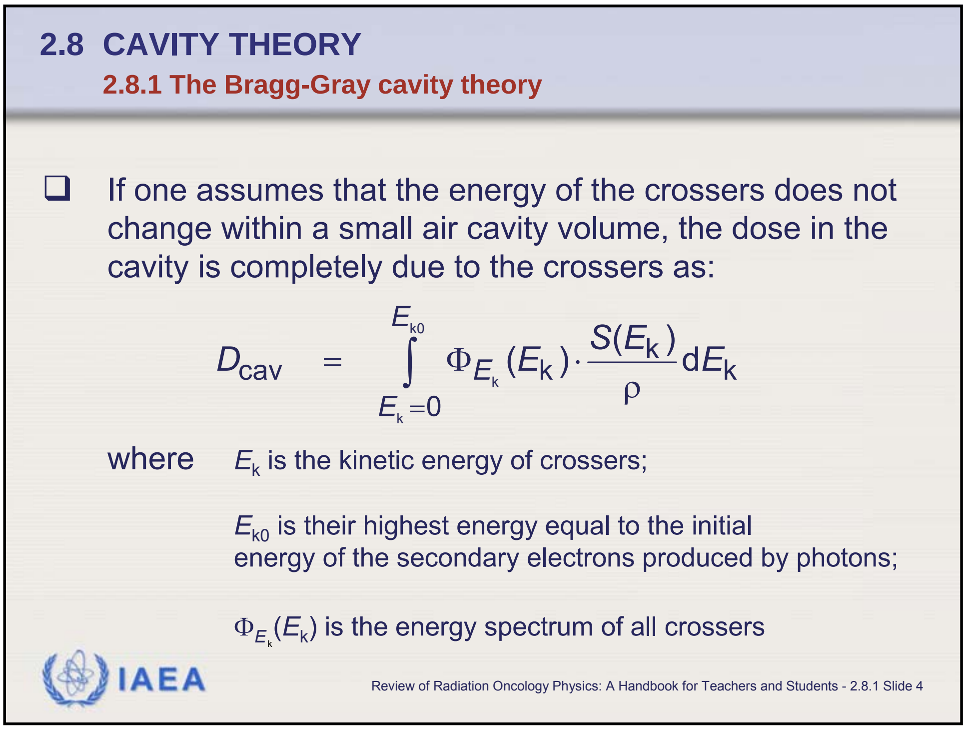


## 2.8 CAVITY THEORY

### 2.8.1 The Bragg-Gray cavity theory

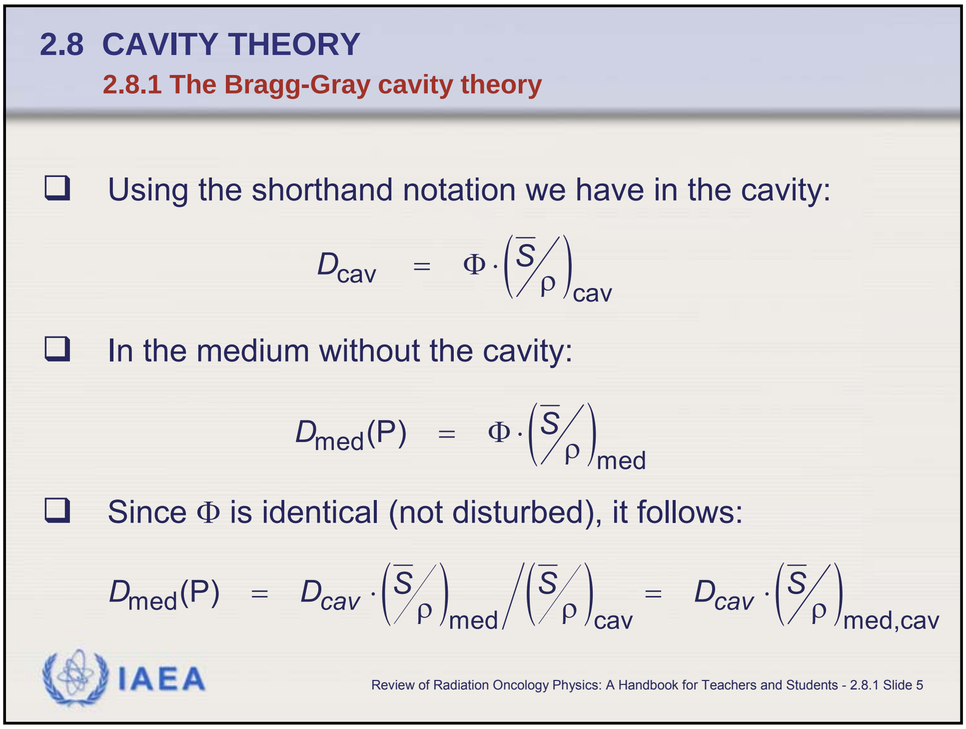
❑ If one assumes that the energy of the crossers does not change within a small air cavity volume, the dose in the cavity is completely due to the crossers as:

$$D_{\text{cav}} = \int_{E_k=0}^{E_{k0}} \Phi_{E_k}(E_k) \cdot \frac{S(E_k)}{\rho} \, dE_k$$

where $E_k$ is the kinetic energy of crossers;

$E_{k0}$ is their highest energy equal to the initial energy of the secondary electrons produced by photons;

$\Phi_{E_k}(E_k)$ is the energy spectrum of all crossers



## 2.8 CAVITY THEORY

### 2.8.1 The Bragg-Gray cavity theory

- ☐ Using the shorthand notation we have in the cavity:

$$D_{\mathrm{cav}} = \Phi \cdot \left( \bar{S}/\rho \right)_{\mathrm{cav}}$$

- ☐ In the medium without the cavity:

$$D_{\mathrm{med}}(\mathrm{P}) = \Phi \cdot \left( \bar{S}/\rho \right)_{\mathrm{med}}$$

- ☐ Since $\Phi$ is identical (not disturbed), it follows:

$$D_{\mathrm{med}}(\mathrm{P}) = D_{cav} \cdot \left( \bar{S}/\rho \right)_{\mathrm{med}} \Big/ \left( \bar{S}/\rho \right)_{\mathrm{cav}} = D_{cav} \cdot \left( \bar{S}/\rho \right)_{\mathrm{med,cav}}$$

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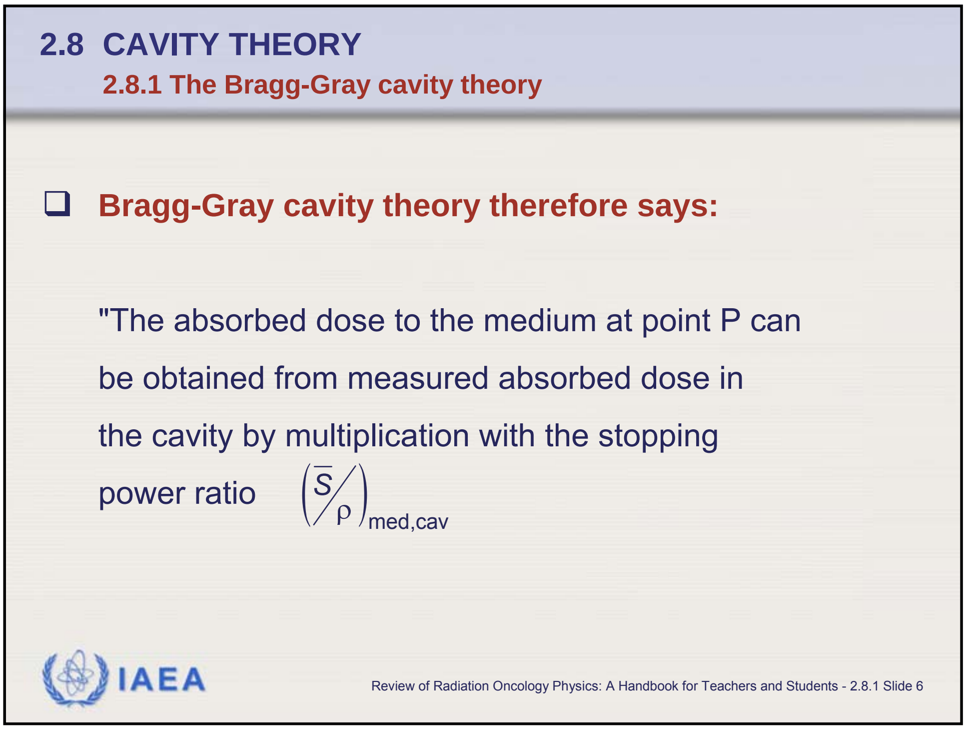

## 2.8 CAVITY THEORY

### 2.8.1 The Bragg-Gray cavity theory

- ☐ **Bragg-Gray cavity theory therefore says:**

"The absorbed dose to the medium at point P can be obtained from measured absorbed dose in the cavity by multiplication with the stopping power ratio $\left( \bar{S}/\rho \right)_{\mathrm{med,cav}}$

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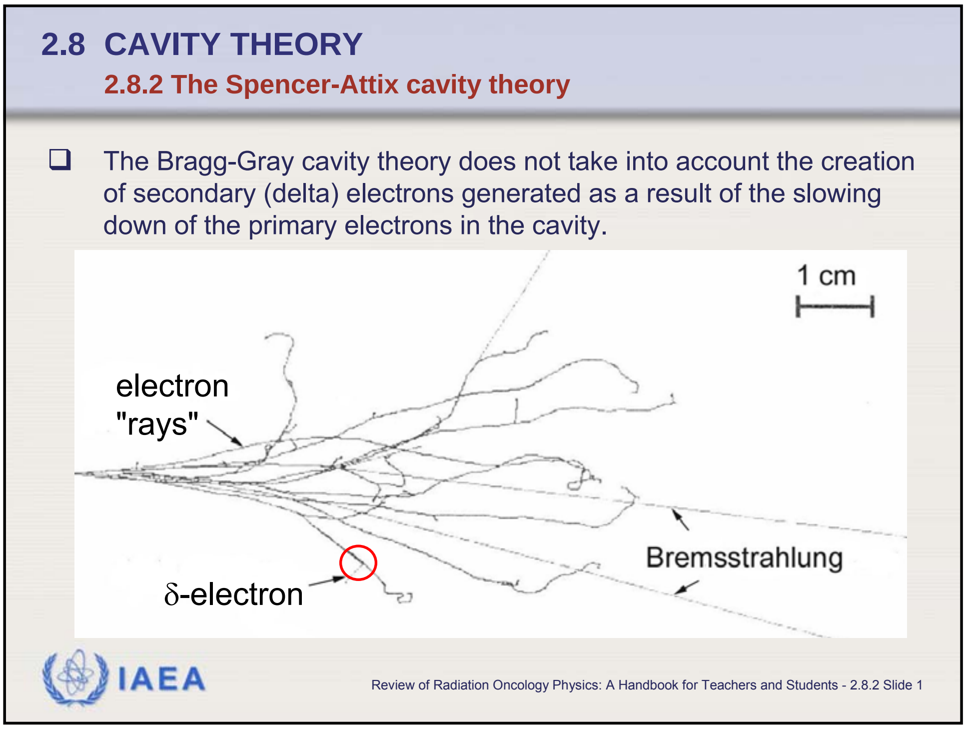
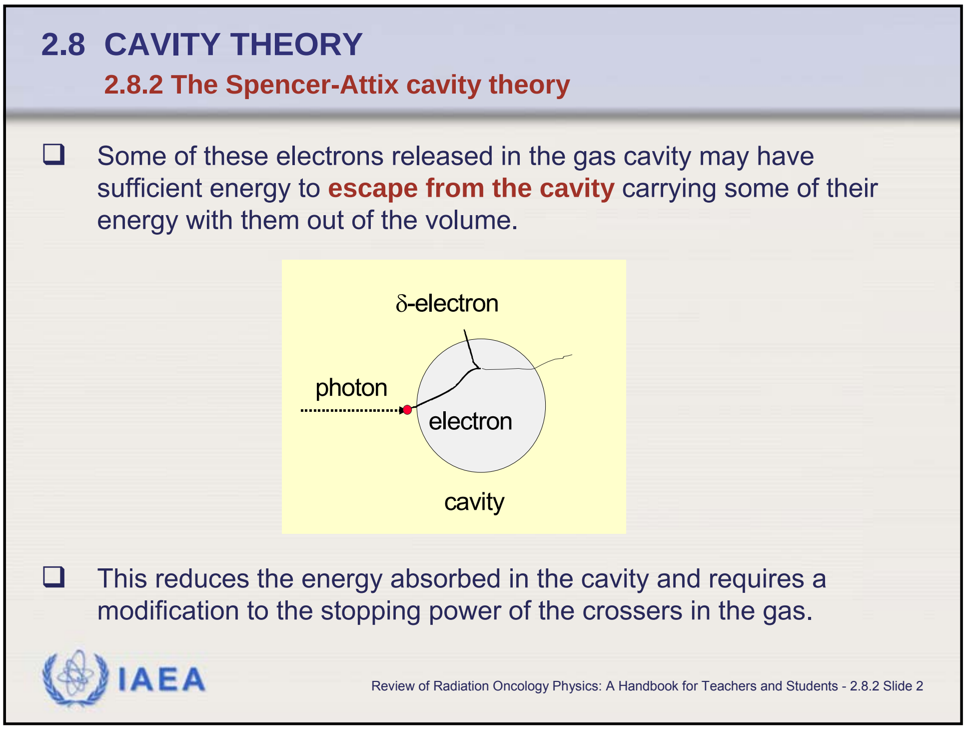


# 2.8 CAVITY THEORY

## 2.8.2 The Spencer-Attix cavity theory

- ❑ The Bragg-Gray cavity theory does not take into account the creation of secondary (delta) electrons generated as a result of the slowing down of the primary electrons in the cavity.



# 2.8 CAVITY THEORY

## 2.8.2 The Spencer-Attix cavity theory

- ❑ Some of these electrons released in the gas cavity may have sufficient energy to **escape from the cavity** carrying some of their energy with them out of the volume.

- ❑ This reduces the energy absorbed in the cavity and requires a modification to the stopping power of the crossers in the gas.

## 2.8 CAVITY THEORY

### 2.8.2 The Spencer-Attix cavity theory

- ❑ This is accomplished in the Spencer-Attix cavity theory by explicitly considering the $\delta$ electrons.
- ❑ Spencer-Attix cavity theory operates under the same two conditions as used in the Bragg-Gray cavity theory.
- ❑ However, these conditions are now applied also to the fluence of the $\delta$ electrons.

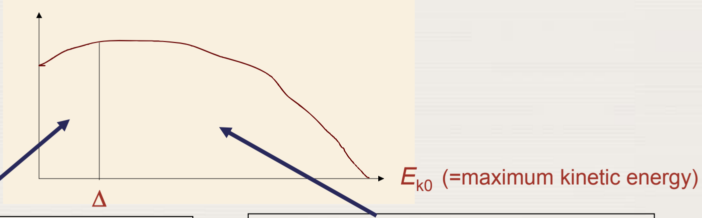
---

## 2.8 CAVITY THEORY

### 2.8.2 The Spencer-Attix cavity theory

**The concept of the Spencer-Attix cavity theory:**

The total secondary electron fluence (crossers and $\delta$ electrons) is divided into two components based on a user-defined **energy threshold $\Delta$**.

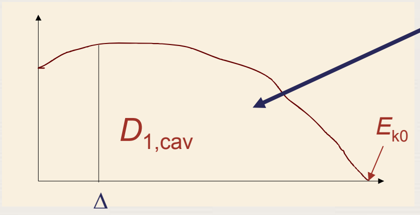


Secondary electrons with kinetic energies $E_k$ less than $\Delta$ are considered "slow" electrons. They deposit their energy locally.

Secondary electrons with energies larger than or equal to $\Delta$ are considered “fast” electrons. They all deposit their energy like crossers.

## 2.8 CAVITY THEORY

### 2.8.2 The Spencer-Attix cavity theory



- ☐ All secondary electrons with energies $E_k > \Delta$ are treated as crossers.
- ☐ It means that such $\delta$ electrons with $E_k > \Delta$ must be included in the entire electron spectrum.

$$D_{1,\mathrm{cav}} = \int_0^{E_{k0}} \Phi_{E_k}^{\delta}(E_k) \cdot \frac{S_{\mathrm{cav}}(E_k)}{\rho} \, dE_k$$

where $\Phi_{E_k}^{\delta}(E_k)$ is now the energy spectrum of

**all electrons including the $\delta$ electrons with $E_k > \Delta$**

---

## 2.8 CAVITY THEORY

### 2.8.2 The Spencer-Attix cavity theory

- ☐ However, this equation

$$D_{1,\mathrm{cav}} = \int_0^{E_{k0}} \Phi_{E_k}^{\delta}(E_k) \cdot \frac{S_{\mathrm{cav}}(E_k)}{\rho} \, dE_k$$

**is not correct because the energy of the $\delta$ electrons is now taken into account twice:**

- as part of the spectrum of electrons
- in the unrestricted stopping power as the energy lost ranging up to the maximum energy lost (including that larger than $\Delta$)

## 2.8 CAVITY THEORY

### 2.8.2 The Spencer-Attix cavity theory

- ☐ Solution of this situation:
  The calculation must refer to the **restricted** mass stopping power:

$$L_\Delta = \frac{dE_\Delta}{dl}$$

$$\Rightarrow \quad D_{1,\text{cav}} = \int_0^{E_{k0}} \Phi_{E_k}^{\delta}(E_k)\cdot\frac{L_{\Delta,\text{cav}}(E_k)}{\rho}\,dE_k$$

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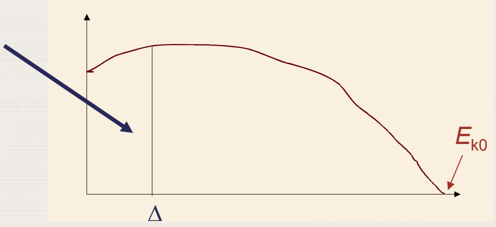
## 2.8 CAVITY THEORY

### 2.8.2 The Spencer-Attix cavity theory



- ☐ Secondary electrons with kinetic energies $K_E < \Delta$ are considered slow electrons. They deposit their energy "locally"
- ☐ "Locally" means that they can be treated as so-called "stoppers". $D_{2,\text{cav}}$ is sometimes called the "track-end term".
- ☐ Energy deposition of "stoppers" **cannot** be described by stopping power.
- ☐ Their energy lost is simply their (local) kinetic energy.

$$\Rightarrow \quad D_{2,\text{cav}} = \textit{energy of stoppers per mass}$$

## 2.8 CAVITY THEORY

### 2.8.2 The Spencer-Attix cavity theory

☐ For practical calculations, the track-end term *TE* was approximated by A. Nahum as:

$$TE = \Phi_{E_k}^{\delta}(\Delta) \cdot \frac{S(\Delta)}{\rho} \cdot \Delta$$

☐ Finally we have:

$$D_{\mathrm{cav}} = \int_{0}^{E_{k0}} \Phi_{E_k}^{\delta}(E_k) \cdot \frac{L_{\Delta,\mathrm{cav}}(E_k)}{\rho} \mathrm{d}E_k + TE$$

---

## 2.8 CAVITY THEORY

### 2.8.2 The Spencer-Attix cavity theory

☐ In the Spencer-Attix cavity theory, the stopping power ratio is therefore obtained by:

$$\left(\bar{S}/\rho\right)_{\mathrm{med,cav}} = \frac{\displaystyle\int_{0}^{E_{k0}} \Phi_{E_k}^{\mathrm{med},\delta}(E_k) \cdot \frac{L_{\Delta,\mathrm{med}}(E_k)}{\rho} \mathrm{d}E_k + \Phi_{E_k}^{\mathrm{med},\delta}(\Delta) \cdot \frac{S_{\mathrm{med}}(\Delta)}{\rho} \cdot \Delta}{\displaystyle\int_{0}^{E_{k0}} \Phi_{E_k}^{\mathrm{cav},\delta}(E_k) \cdot \frac{L_{\Delta,\mathrm{cav}}(E_k)}{\rho} \mathrm{d}E_k + \Phi_{E_k}^{\mathrm{cav},\delta}(\Delta) \cdot \frac{S_{\mathrm{cav}}(\Delta)}{\rho} \cdot \Delta}$$

## 2.8 CAVITY THEORY

### 2.8.3 Considerations in the application of cavity theory to ionization chamber calibration and dosimetry protocols

- The value of the energy threshold Δ is set 10 keV.
- In the context of cavity theories, the sensitive volume of the dosimeter can be identified as the “cavity”, which may contain a gaseous, liquid or solid medium (e.g. TLD).
- In ionization chambers, air is used as the sensitive medium, since it allows a relatively simple electrical means for collection of charges released in the sensitive medium by radiation.

## 2.8 CAVITY THEORY

### 2.8.3 Considerations in the application of cavity theory to ionization chamber calibration and dosimetry protocols

- In current dosimetry concepts, the ionization chamber is used
  - in a phantom
  - **without** a build-up material.
- Typical thicknesses of the chamber wall are much thinner than the range of the secondary electrons.
- Therefore, the proportion of the cavity dose due to electrons generated in the phantom greatly exceeds the dose contribution from the wall.
- Hence, the **phantom** medium serves as the medium and the wall is treated as a perturbation.

# 2.8 CAVITY THEORY

## 2.8.3 Considerations in the application of cavity theory to ionization chamber calibration and dosimetry protocols

- ☐ Taking into account all further small perturbations, the dose in the medium is determined with a thin-walled ionization chamber in a high energy photon or electron beam by:

$$D_{med} = \frac{Q}{m} \cdot \left( \frac{W_{gas}}{e} \right) \cdot s_{med,gas}^{SA} \cdot p_{fl} \cdot p_{dis} \cdot p_{wall} \cdot p_{cel}$$

where

| | |
|---|---|
| $s_{med,gas}^{SA}$ | is the Spencer-Attix stopping power ratio |
| $W_{gas}$ | is the average energy expended in air per ion pair formed |
| $p_{fl}$ | is the electron fluence perturbation correction factor; |
| $p_{dis}$ | is the correction factor for displacement of the effective measurement point; |
| $p_{wall}$ | is the wall correction factor; |
| $p_{cel}$ | is the correction factor for the central electrode; |

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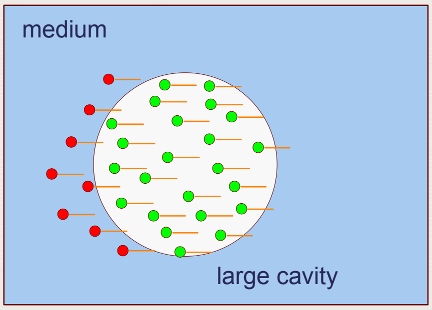
# 2.8 CAVITY THEORY

## 2.8.4 Large cavities in photon beams

- ☐ A large cavity is a cavity such that the dose contribution from secondary electrons (= — ) originating outside the cavity (= ● ) can be ignored when compared with the contribution of electrons created by photon interactions within the cavity (= ● ).



## 2.8 CAVITY THEORY

### 2.8.4 Large cavities in photon beams

- For a large cavity the ratio of dose cavity to medium is calculated as the ratio of the collision kerma in the cavity to the medium and is therefore equal to the ratio of the average mass-energy absorption coefficients, cavity to medium:

$$D_{\text{med}} = D_{\text{gas}} \cdot \left( \frac{\overline{\mu}_{\text{en}}}{\rho} \right)_{\text{med,gas}}$$

where the mass-energy absorption coefficients have been averaged over the photon fluence spectra in the medium (numerator) and in the cavity gas (denominator).

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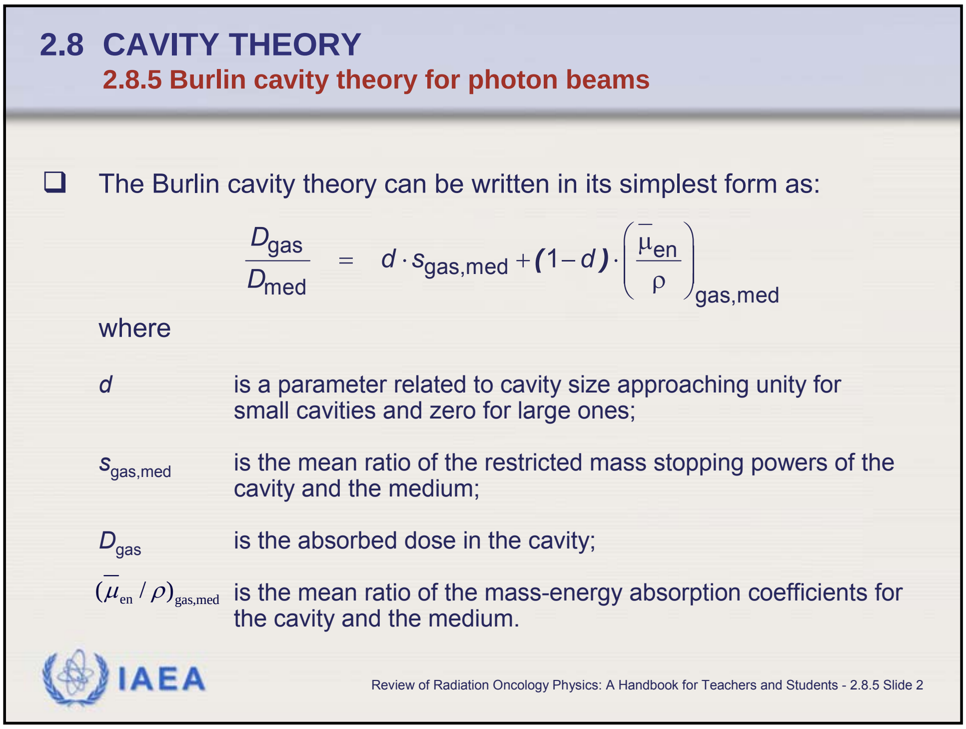
## 2.8 CAVITY THEORY

### 2.8.5 Burlin cavity theory for photon beams

- Burlin extended the Bragg-Gray and Spencer-Attix cavity theories to cavities of **intermediate** dimensions by introducing the large cavity limit to the Spencer-Attix equation using a **weighting technique**.
- This was introduced on a purely phenomenological basis.
- He provided a formalism to calculate the value of the weighting parameter.



## 2.8 CAVITY THEORY

### 2.8.5 Burlin cavity theory for photon beams

- The Burlin cavity theory can be written in its simplest form as:

$$\frac{D_{\text{gas}}}{D_{\text{med}}} = d \cdot s_{\text{gas,med}} + (1-d) \cdot \left(\frac{\bar{\mu}_{\text{en}}}{\rho}\right)_{\text{gas,med}}$$

where

| | |
|---|---|
| $d$ | is a parameter related to cavity size approaching unity for small cavities and zero for large ones; |
| $s_{\text{gas,med}}$ | is the mean ratio of the restricted mass stopping powers of the cavity and the medium; |
| $D_{\text{gas}}$ | is the absorbed dose in the cavity; |
| $(\bar{\mu}_{\text{en}}/\rho)_{\text{gas,med}}$ | is the mean ratio of the mass-energy absorption coefficients for the cavity and the medium. |

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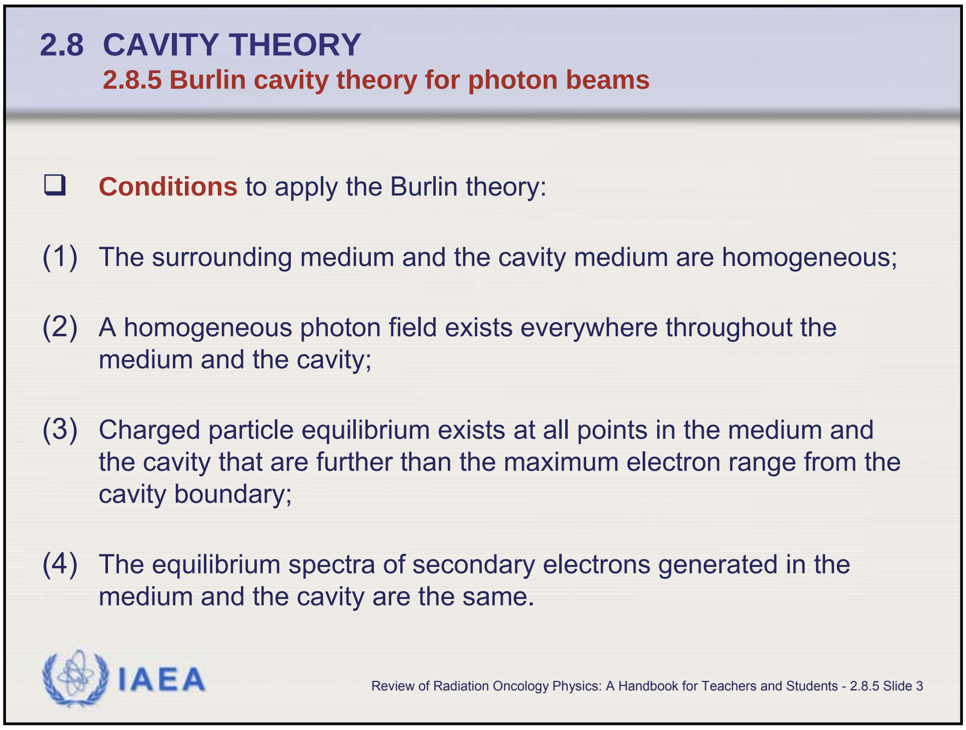

## 2.8 CAVITY THEORY

### 2.8.5 Burlin cavity theory for photon beams

- **Conditions** to apply the Burlin theory:

(1) The surrounding medium and the cavity medium are homogeneous;

(2) A homogeneous photon field exists everywhere throughout the medium and the cavity;

(3) Charged particle equilibrium exists at all points in the medium and the cavity that are further than the maximum electron range from the cavity boundary;

(4) The equilibrium spectra of secondary electrons generated in the medium and the cavity are the same.

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# 2.8 CAVITY THEORY
## 2.8.5 Burlin cavity theory for photon beams

- **How to get the the weighting parameter *d* in this theory?**
- Burlin provided the following method:
  - *d* is expressed as the average value of the electron fluence reduction in the medium.
  - Consistent with experiments with β-sources he proposed that on average the electron fluence in the medium $\Phi^{e}_{med}$ decays exponentially.
  - The value of the weighting parameter *d* in conjunction with the stopping power ratio can be calculated as:

$$d = \frac{\int_0^L \Phi^{e}_{med} e^{-\beta l} dl}{\int_0^L \Phi^{e}_{med} dl} = \frac{1 - e^{-\beta L}}{\beta L}$$

---

# 2.8 CAVITY THEORY
## 2.8.5 Burlin cavity theory for photon beams

- In this theory, β is an effective electron fluence attenuation coefficient that quantifies the reduction in particle fluence from its initial medium fluence value through a cavity of average length L.
- For convex cavities and isotropic electron fluence distributions, *L* can be calculated as *4V/S* where *V* is the cavity volume and *S* its surface area.
- Burlin described the build-up of the electron fluence Φ inside the cavity using a similar, complementary equation:

$$1 - d = \frac{\int_0^L \Phi^{e}_{gas} \cdot (1 - e^{-\beta l}) dl}{\int_0^L \Phi^{e}_{gas} dl} = \frac{\beta L - 1 + e^{-\beta L}}{\beta L}$$

## 2.8 CAVITY THEORY

### 2.8.5 Burlin cavity theory for photon beams

- Burlin’s theory is consistent with the fundamental constraint of cavity theory that, the weighting factors of both terms add up to unity (*i.e.*, *d* and 1-*d*).
- It had relative success in calculating ratios of absorbed dose for some types of intermediate cavities.
- More generally, however, Monte Carlo calculations show that, when studying ratios of directly calculated absorbed doses in the cavity to absorbed dose in the medium as a function of cavity size, the weighting method is too simplistic and additional terms are necessary to calculate dose ratios for intermediate cavity sizes.
- **For these and other reasons, the Burlin cavity theory is no longer used in practice.**

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## 2.8 CAVITY THEORY

### 2.8.6 Stopping power ratios

- For high energy photons and electrons, the stopping power ratio as defined by:

$$s_{\text{med1,med2}} = \left(\frac{\bar{S}}{\rho}\right)_{\text{med1}} \Big/ \left(\frac{\bar{S}}{\rho}\right)_{\text{med2}}$$

is the important link to perform an absolute measurement of absorbed dose in a medium med1 with a dosimeter made of a medium med2.

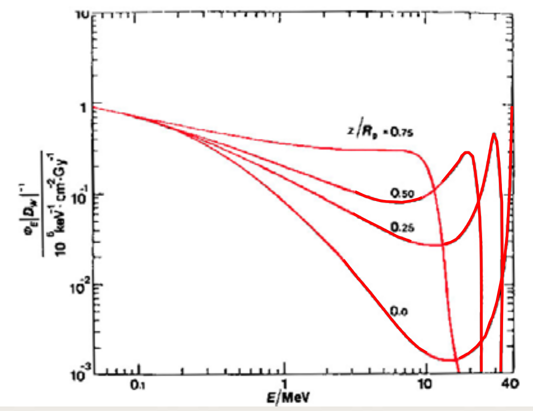
## 2.8 CAVITY THEORY

### 2.8.6 Stopping power ratios

- ☐ It is also in particular relevant in performing accurate **relative** measurements of absorbed dose in a phantom where the energy of the electrons changes significantly.

**Example from ICRU 35:**
Energy spectrum of electrons with of 40 MeV initial energy in a water phantom at **different depth z** (expressed by $z/R_{p)}$

Values were normalized to that at surface for 40 MeV .



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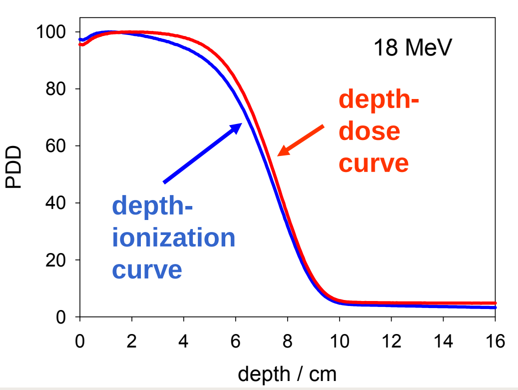
## 2.8 CAVITY THEORY

### 2.8.6 Stopping power ratios

- ☐ The measurement of the relative dose in air changes with depth with an ionization chambers always provide a **depth-ionization** curve.

- ☐ The **depth-ionization curve** of electrons differs from the **depth-dose curve** by the water-to-air stopping power ratio.



# 2.8 CAVITY THEORY

## 2.8.6 Stopping power ratios

- ❑ As shown in the previous slides, the Spencer-Attix ratio of restricted collision stopping powers are required for this.
- ❑ However, due to the energy distribution of electrons at each point along the depths of measurement, one **CANNOT** use directly the stopping power ratios for monoenergetic electrons.
- ❑ Instead of, one must determine them for the energy distribution of electrons at realistic linac beams.

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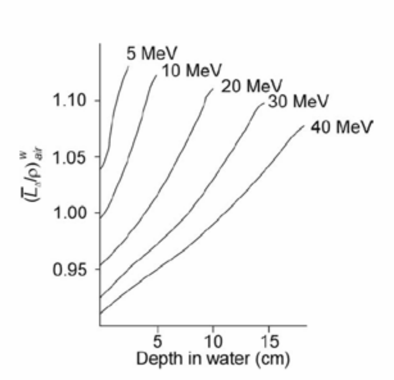
# 2.8 CAVITY THEORY

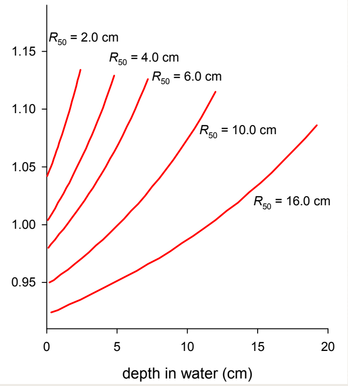
## 2.8.6 Stopping power ratios

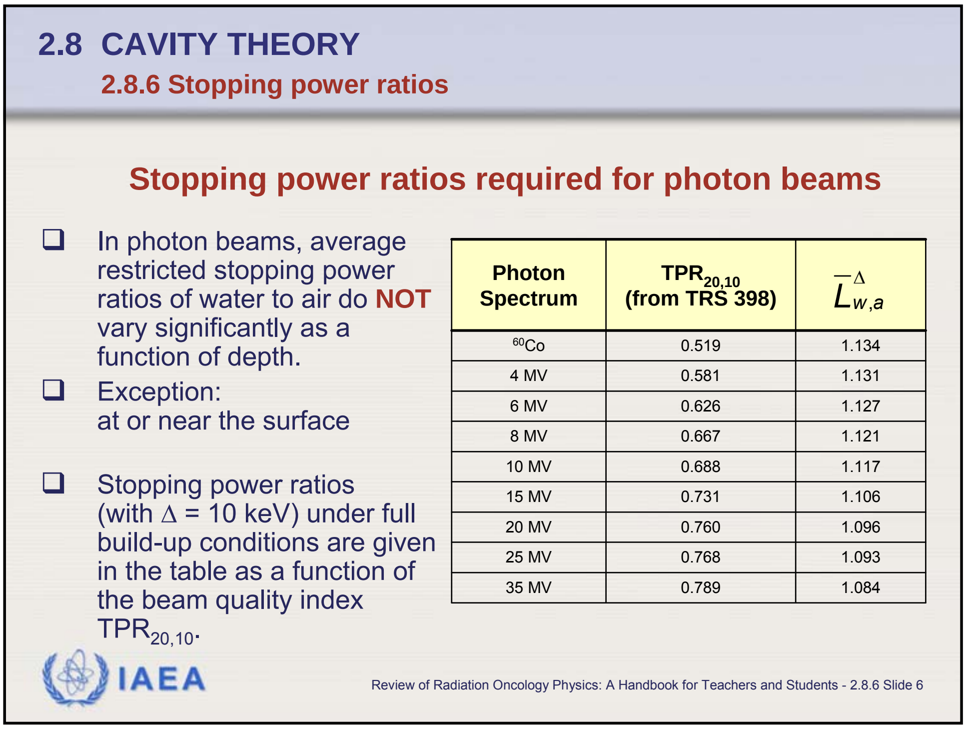
Restricted stopping power ratios ($\Delta$ = 10 keV) of water to air for electron beams as a function of depth in water.

**for mono-energetic electrons**



**for realistic linac beams (from TRS 398)**





# 2.8 CAVITY THEORY

## 2.8.6 Stopping power ratios

### Stopping power ratios required for photon beams

- In photon beams, average restricted stopping power ratios of water to air do **NOT** vary significantly as a function of depth.
- Exception:
  at or near the surface
- Stopping power ratios (with $\Delta$ = 10 keV) under full build-up conditions are given in the table as a function of the beam quality index $TPR_{20,10}$.

| **Photon Spectrum** | **$TPR_{20,10}$ (from TRS 398)** | $\overline{L}^{\Delta}_{w,a}$ |
|---|---|---|
| $^{60}$Co | 0.519 | 1.134 |
| 4 MV | 0.581 | 1.131 |
| 6 MV | 0.626 | 1.127 |
| 8 MV | 0.667 | 1.121 |
| 10 MV | 0.688 | 1.117 |
| 15 MV | 0.731 | 1.106 |
| 20 MV | 0.760 | 1.096 |
| 25 MV | 0.768 | 1.093 |
| 35 MV | 0.789 | 1.084 |

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